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DYNAMIC BEHAVIORS OF MAY TYPE COOPERATIVE SYSTEM WITH MICHAELIS-MENTEN TYPE HARVESTING[*†]

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Abstract

Traditional May type cooperative model incorporating Michaelis-Menten type harvesting is proposed and studied in this paper. Sufficient conditions which ensure the extinction of the first species and the existence of a unique globally attractive positive equilibrium are obtained, respectively. Numeric simulations are carried out to show the feasibility of the main results.

Keywords global attractivity; May type cooperative system; Michaelis-Menten type harvesting; iterative method

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1 Introduction

The aim of this paper is to investigate the dynamic behaviors of the following May type cooperative model incorporating Michaelis-Menten type harvesting

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - \frac{Eqx}{m_1 E + m_2 x}, \\ \dot{y} &= y\Big(r_2 - b_2 y - \frac{a_2 y}{x + k_2}\Big), \end{aligned} \tag{1.1}$$

where x and y denote the densities of two populations at time t. The parameters $r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q, m_1, m_2$ are all positive constants.

During the last decade, many scholars [1-30] investigated the dynamic behaviors of the cooperative system. Yang, Miao, Chen et al [4], Yang and Li [9], Chen, Chen, Li [10], Chen and Xie [11], Han, Xie and Chen [12], Chen and Xie [13], Han, Chen, Xie et al [14], Chen, Yang, Chen et al [15] studied the influence of feedback controls

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on the cooperative system; May [1], Xie, Chen and Xue [2], Chen, Wu and Xie [3], Xie, Chen, Yang et al [6], Yang, Xie and Chen [7], Chen, Xie and Chen [8], Chen, Xue, Lin et al [16], Wu and Lin [20], Li, Chen, Chen et al [23], Lin [25], Deng and Huang [26], Lei [27, 28], Chen [29, 30] studied the stability property of the equilibria of cooperative or commensalism model; Chen, Chen and Li [10], Chen and Xie [11], Han, Xie and Chen [12], Chen and Xie [13], Chen, Yang, Chen et al [15], Yang, Xie, Chen et al [19] investigated the persistent property of the cooperative system; Lin [24], Chen [30], Wu [21] investigated the influence of Allee effect on the cooperative system or commensalism system; Xue, Xie and Chen [5], Yang, Xie and Chen [18], Muhammadhaji and Teng [22] investigated the periodic solution or almost periodic solution of the cooperative system.

However, only recently has it attracted the attention of scholars([2, 3, 25–27, 29]) to investigate the influence of harvesting on the cooperative or commensalism model. Xie, Chen and Xue [2] studied the following cooperative system incorporating linear harvesting to the first species

$$\begin{aligned}\dot{x} &= x\Big(r_1 - b_1x - \frac{a_1x}{y+k_1}\Big) - Eqx,\\ \dot{y} &= y\Big(r_2 - b_2y - \frac{a_2y}{x+k_2}\Big),\end{aligned}\tag{1.2}$$

where x and y denote the densities of two populations at time t. The parameters $r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q$ are all positive constants. They showed that if $r_1 > Eq$ holds, then the unique positive equilibrium $E^*(x^*, y^*)$ of system (1.2) is globally attractive.

Lei [27] studied the dynamic behaviors of the following non-selective harvesting May cooperative system incorporating partial closure for the populations

$$\begin{aligned}\dot{x} &= x\Big(r_1 - b_1x - \frac{a_1x}{y+k_1}\Big) - Eq_1mx,\\ \dot{y} &= y\Big(r_2 - b_2y - \frac{a_2y}{x+k_2}\Big) - Eq_2my,\end{aligned}\tag{1.3}$$

where x and y denote the densities of two populations at time t. The parameters r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q_1 and q_2 are all positive constants, E is the combined fishing effort used to harvest and $m\,(0 < m < 1)$ is the fraction of the stock available for harvesting. His study showed that the intrinsic growth rate and the fraction of the stocks for the harvesting plays crucial role on the dynamic behaviors of the system, all of the four equilibria maybe globally attractive under some suitable assumption.

It brings to our attention that in system (1.2) and (1.3), the authors chose the linear harvesting. Suck kind of harvesting embodies several unrealistic features and limitations. For example, in system (2.1), the authors took $h(E,x) = qEx$ as the fishing term, where E denotes effort. One could see that h tends to infinity as the effort E tends to infinity if the population x is finite and fixed, or as the population x tends to infinity if the effort E is finite and fixed. To overcome this drawback, recently, many scholars [30–33] argued that the nonlinear harvesting, or named as Michaelis-Menten type harvesting is more suitable, it is more appropriate to describe the fishing process of human being. Chen [30] incorporated the Michaelis-Menten type harvesting term to the first species of the commensalism model, and studied the following model:

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= r_1x\Big(1 - \frac{x}{K_1} + \alpha\frac{y}{K_1}\Big) - \frac{qEx}{m_1E + m_2x},\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= r_2y\Big(1 - \frac{y}{K_2}\Big),\end{aligned} \tag{1.4}$$

where r_1, r_2, K_1, K_2, α, q, E, m_1, m_2 are all positive constants, r_1, r_2, K_1, K_2, α have the same meaning as those of system (1.1), E is the fishing effort used to harvest and q is the catchablity coefficient, m_1 and m_2 are suitable constants. In system (1.4), where the harvesting term is $h(E,x) = qEx/(m_1E + m_2x)$, q is the catchability coefficient, E is the external effort devoted to harvesting, one could see that $\lim\limits_{E\to+\infty} h(E,x) = qx/m_1$ and $\lim\limits_{x\to+\infty} h(E,x) = qE/m_2$. Such an assumption obviously overcome the drawback of the linear one.

It brings to our attention that **to this day, still no scholars propose and study the cooperative system withe Michaelis-Menten type harvesting**. This motivates us to propose system (1.1), that is, we incorporate the Michaelis-Menten type harvesting to the traditional May cooperative system [1]. As far as system (1.1) is concerned, the most important thing is to study the extinction and persistent property of the system.

We will investigate the extinction property in the next section, and investigate the stability property of the positive equilibrium in Section 3, finally we end this paper by a briefly discussion.

2 Extinction of the First Species

As a direct corollary of Lemma 2.2 of Chen [17], we have:

Lemma 2.1 *If $a>0$, $b>0$ and $\dot{x}\ge x(b-ax)$, when $t\ge 0$ and $x(0)>0$, we have*

$$\liminf_{t\to+\infty} x(t) \ge \frac{b}{a}.$$

If $a > 0$, $b > 0$ and $\dot{x} \le x(b - ax)$, when $t \ge 0$ and $x(0) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \le \frac{b}{a}.$$

Concerned with the extinction of the first species, we have the following result.

Theorem 2.1 *Assume that*

$$r_1 < \frac{qE}{m_1E + m_2F}, \tag{2.1}$$

where

$$F = \frac{r_1}{b_1 + \dfrac{a_1}{\dfrac{r_2}{b_2} + k_1}}.$$

Then the first species will be driven to extinction, that is,

$$\lim_{t\to+\infty} x(t) = 0.$$

Proof Condition (2.1) implies that for enough small positive constant $\varepsilon > 0$,

$$r_1 < \frac{qE}{m_1E + m_2F(\varepsilon)} \tag{2.2}$$

holds, where

$$F(\varepsilon) = \frac{r_1}{b_1 + \dfrac{a_1}{\dfrac{r_2}{b_2} + \varepsilon + k_1}} + \varepsilon.$$

Now from the second equation of system (1.1), we have

$$\dot{y} \le y\big(r_2 - b_2y\big). \tag{2.3}$$

Applying Lemma 2.1 to (2.3) leads to

$$\limsup_{t\to+\infty} y(t) \le \frac{r_2}{b_2}. \tag{2.4}$$

Therefore, for $\varepsilon > 0$ small enough which satisfies (2.2), there exists a $T_1 > 0$ such that

$$y(t) \le \frac{r_2}{b_2} + \varepsilon \quad \text{for all } t \ge T_1. \tag{2.5}$$

For $t > T_1$, from (2.5) and the first equation of system (1.1), we have

$$\begin{aligned}\dot{x} &\le x\left(r_1 - b_1x - \frac{a_1x}{\dfrac{r_2}{b_2} + \varepsilon + k_1}\right) - \frac{Eqx}{m_1E + m_2x}\\ &\le x\left(r_1 - b_1x - \frac{a_1x}{\dfrac{r_2}{b_2} + \varepsilon + k_1}\right).\end{aligned} \tag{2.6}$$

Applying Lemma 2.1 to (2.6) leads to

$$\lim_{t\to+\infty} x(t) \le \frac{r_1}{F_{11}}, \tag{2.7}$$

where

$$F_{11} = b_1 + \frac{a_1}{\frac{r_2}{b_2} + \varepsilon + k_1}.$$

Hence, there exists a $T_2 > T_1$ such that

$$x(t) < \frac{r_1}{F_{11}} + \varepsilon \overset{\text{def}}{=} F(\varepsilon) \quad \text{for all } t > T_2. \tag{2.8}$$

For $t > T_2$, again, from the first equation of system (1.1), we have

$$\begin{aligned} \dot{x} &\le x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - \frac{Eqx}{m_1 E + m_2 x} \\ &\le x\Big(r_1 - \frac{Eq}{m_1 E + m_2 x}\Big) \\ &\le x\Big(r_1 - \frac{Eq}{m_1 E + m_2 F(\varepsilon)}\Big). \end{aligned} \tag{2.9}$$

Hence,

$$x(t) \le x(T_2) \exp\Big\{\Big(r_1 - \frac{Eq}{m_1 E + m_2 F(\varepsilon)}\Big)(t - T_2)\Big\}. \tag{2.10}$$

It then immediately follows from (2.2) that

$$\lim_{t\to+\infty} x(t) = 0.$$

This ends the proof of Theorem 2.1.

Remark 2.1 Condition (2.1) seems a little complicated, since we here try to incorporate the influence of the second species, however, from the proof of Theorem 2.1, in (2.6), from the first equation of system (1.1), we could also have

$$\dot{x} \le x\big(r_1 - b_1 x\big). \tag{2.11}$$

Applying Lemma 2.1 to (2.11), one has

$$\limsup_{t\to+\infty} x(t) \le \frac{r_1}{b_1}. \tag{2.12}$$

From (2.12), with some minor revise of (2.8)-(2.10), we could establish the following more stronger but seems concise result.

Corollary 2.1 *Assume that*

$$r_1 < \frac{qE}{mE + m_1 \frac{r_1}{b_1}}. \tag{2.13}$$

Then the first species will be driven to extinction, that is,

$$\lim_{t\to+\infty} x(t) = 0.$$

Remark 2.2 For the system without fishing, that is, $q = 0$ in system (1.1), May [1] showed that the system admits a unique globally attractive positive equilibrium. That is, two species could be coexist in a stable state. Theorem 2.1 and Corollary 2.1 show the over harvesting of the first species (that is, q in system (1.1) is too large), then despite the cooperation between the species, the first species will still be driven to extinction.

Concerned with the stability of the rest species y, we have the following result.

Theorem 2.2 *Assume that* (2.1) *or* (2.13) *holds, then*

$$\lim_{t\to+\infty} y(t) = \frac{r_2}{b_2 + \dfrac{a_2}{k_2}}. \tag{2.14}$$

Proof It follows from the second equation of system (1.1) that

$$\dot{y} \geq y\Big(r_2 - b_2 y - \frac{a_2 y}{k_2}\Big). \tag{2.15}$$

Applying Lemma 2.1 to (2.15) leads to

$$\liminf_{t\to+\infty} y(t) \geq \frac{r_2}{b_2 + \dfrac{a_2}{k_2}}. \tag{2.16}$$

On the other hand, under assumption (2.1) or (2.13), from Theorem 2.1 and Corollary 2.1, we know that the first species in system (1.1) will be driven to extinction. That is, for any enough small positive constant $\varepsilon > 0$, there exists an enough large T such that

$$x(t) < \varepsilon \quad \text{for all } t \geq T. \tag{2.17}$$

From (2.17) and the second equation of system (1.1), we have

$$\dot{y} \leq y\Big(r_2 - b_2 y - \frac{a_2 y}{\varepsilon + k_2}\Big). \tag{2.18}$$

Hence, from Lemma 2.1, we have

$$\limsup_{t\to+\infty} y(t) \leq \frac{r_2}{b_2 + \dfrac{a_2}{\varepsilon + k_2}}. \tag{2.19}$$

Since ε is an enough small positive constant, setting $\varepsilon \to 0$ in (2.19) leads to

$$\limsup_{t\to+\infty} y(t) \leq \frac{r_2}{b_2 + \dfrac{a_2}{k_2}}. \tag{2.20}$$

Combining (2.16) with (2.20), we have

$$\lim_{t\to+\infty} y(t) = \frac{r_2}{b_2 + \dfrac{a_2}{k_2}}. \tag{2.21}$$

This ends the proof of Theorem 2.2.

3 Stability of the Positive Equilibrium

First, let's investigate the existence of the positive equilibrium of system (1.1).

Theorem 3.1 *Assume that*

$$b_1 k_2 > r_1 > \frac{q}{m_1} \tag{3.1}$$

holds, then system (1.1) *admits a unique positive equilibrium.*

Proof The positive equilibrium of system (1.1) satisfies the equations

$$\begin{aligned} & r_1 - b_1 x - \frac{a_1 x}{y + k_1} - \frac{Eq}{m_1 E + m_2 x} = 0, \\ & r_2 - b_2 y - \frac{a_2 y}{x + k_2} = 0. \end{aligned} \tag{3.2}$$

From the second equation, we have

$$y = \frac{r_2\ (x + k_2)}{b_2\, k_2 + b_2\, x + a_2}. \tag{3.3}$$

Substituting (3.3) into the first equation of system (3.2) and by simplifying, we finally obtain

$$A_1 x^2 + A_2 x + A_3 = 0, \tag{3.4}$$

where

$$\begin{aligned} A_1 = {} & E b_1 b_2 k_1 m_1 + E a_1 b_2 m_1 + E b_1 m_1 r_2 + a_1 b_2 k_2 m_2 + a_2 b_1 k_1 m_2 \\ & + a_1 a_2 m_2 + m_2 (b_2 k_1 + r_2)(b_1 k_2 - r_1) \\ A_2 = {} & E b_1 b_2 k_1 k_2 m_1 + E a_1 b_2 k_2 m_1 + E a_2 b_1 k_1 m_1 + E b_1 k_2 m_1 r_2 \\ & - E b_2 k_1 m_1 r_1 - b_2 k_1 k_2 m_2 r_1 + E a_1 a_2 m_1 + E b_2 k_1 q \\ & - E m_1 r_1 r_2 - a_2 k_1 m_2 r_1 - k_2 m_2 r_1 r_2 + E q r_2, \\ A_3 = {} & -E (m_1 r_1 - q)(b_2 k_1 k_2 + a_2 k_1 + r_2 k_2). \end{aligned} \tag{3.5}$$

Under the assumption of Theorem 3.1, $A_1 > 0$ and $A_3 < 0$, hence, (3.4) admits a unique positive solution

$$x^* = \frac{-A_2 + \sqrt{A_2^2 - 4A_1 A_3}}{2A_1}. \tag{3.6}$$

Consequently, system (1.1) admits a unique positive equilibrium $E(x^*, y^*)$, where x^* is defined by (3.6) and

$$y^* = \frac{r_2 (x^* + k_2)}{b_2 k_2 + b_2 x^* + a_2}. \tag{3.7}$$

This ends the proof of Theorem 3.1.

Remark 3.1 Condition $b_1k_2 > r_1$ could be replaced by some restrictions but more complex condition

$$A_1 > 0,$$

where A_1 is defined by (3.5).

Now we are in the position of stating the stability property of the positive equilibrium.

Theorem 3.2 *Assume that* (3.1) *holds, then system* (1.1) *admits a unique positive equilibrium* $E(x^*, y^*)$, *which is globally attractive.*

Proof By the first equation of system (1.1), we have

$$\dot{x}(t) \le x(t)(r_1 - b_1 x(t)).$$

From Lemma 2.1, it follows that

$$\limsup_{t\to+\infty} x(t) \le \frac{r_1}{b_1}.$$

Hence, for enough small $\varepsilon > 0$, without loss of generality, we may assume that

$$\varepsilon < \frac{1}{2}\min\left\{\frac{r_2}{b_2 + \dfrac{a_2}{k_2}}, \frac{r_1 - \dfrac{q}{m_1}}{b_1 + \dfrac{a_1}{k_1}}\right\}. \tag{3.8}$$

It follows from (3.8) that there exists a $T_1' > 0$ such that

$$x(t) < \frac{r_1}{b_1} + \varepsilon \overset{\text{def}}{=} M_1^{(1)} \quad \text{for all } t > T_1'. \tag{3.9}$$

Similarly, for above $\varepsilon > 0$, it follows from the second equation of system (1.1) that there exists a $T_1 > T_1'$ such that

$$y(t) < \frac{r_2}{b_2} + \varepsilon \overset{\text{def}}{=} M_2^{(1)} \quad \text{for all } t > T_1. \tag{3.10}$$

(3.9) and (3.10) together with the first equation of system (1.1) lead to

$$\begin{aligned}\dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - \frac{Eqx}{m_1E + m_2x}\\ &\le x\Big(r_1 - \frac{Eq}{m_1E + m_2M_1^{(1)}} - b_1x - \frac{a_1x}{M_2^{(1)} + k_1}\Big) \quad \text{for all } t > T_1.\end{aligned} \tag{3.11}$$

Therefore, by Lemma 2.1, we have

$$\limsup_{t\to+\infty} x(t) \le \frac{r_1 - \dfrac{Eq}{m_1E + m_2M_1^{(1)}}}{b_1 + \dfrac{a_1}{M_2^{(1)} + k_1}}. \tag{3.12}$$

That is, for $\varepsilon > 0$ which satisfies (3.8), there exists a $T_2' > T_1$ such that

$$x(t) < \frac{r_1 - \dfrac{Eq}{m_1 E + m_2 M_1^{(1)}}}{b_1 + \dfrac{a_1}{M_2^{(1)} + k_1}} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_1^{(2)} > 0 \quad \text{for all } t > T_2'. \tag{3.13}$$

It follows from (3.9) and the second equation of system (1.1) that

$$\dot{y} \le y\Big(r_2 - b_2 y - \frac{a_2 y}{M_1^{(1)} + k_2}\Big). \tag{3.14}$$

Applying Lemma 2.1 to (3.14) leads to

$$\limsup_{t\to+\infty} y(t) \le \frac{r_2}{b_2 + \dfrac{a_2}{M_1^{(1)} + k_2}}.$$

Hence, for $\varepsilon > 0$ which satisfies (3.8), there exists a $T_2 > T_2'$ such that

$$y(t) < \frac{r_2}{b_2 + \dfrac{a_2}{M_1^{(1)} + k_2}} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2^{(2)} > 0 \quad \text{for all } t > T_2. \tag{3.15}$$

Noting that

$$r_1 - \frac{Eq}{m_1 E + m_2 M_1^{(1)}} < r_1, \qquad \frac{a_1}{M_2^{(1)} + k_1} > 0, \qquad \frac{a_2}{M_1^{(1)} + k_2} > 0,$$

it immediately follows that

$$\begin{aligned} M_1^{(2)} &= \frac{r_1 - \dfrac{Eq}{m_1 E + m_2 M_1^{(1)}}}{b_1 + \dfrac{a_1}{M_2^{(1)} + k_1}} + \frac{\varepsilon}{2} < \frac{r_1}{b_1} + \varepsilon = M_1^{(1)}; \\ M_2^{(2)} &= \frac{r_2}{b_2 + \dfrac{a_2}{M_1^{(1)} + k_2}} + \frac{\varepsilon}{2} < \frac{r_2}{b_2} + \varepsilon = M_2^{(1)}. \end{aligned} \tag{3.16}$$

From the first equation of system (1.1) we have

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - \frac{Eqx}{m_1 E + m_2 x} \\ &\ge x\Big(r_1 - \frac{q}{m_1} - b_1 x - \frac{a_1 x}{k_1}\Big) \quad \text{for all } t > T_2. \end{aligned} \tag{3.17}$$

Applying Lemma 2.1 to (3.17) leads to

$$\liminf_{t\to+\infty} x(t) \ge \frac{r_1 - \dfrac{q}{m_1}}{b_1 + \dfrac{a_1}{k_1}}. \tag{3.18}$$

Hence, for $\varepsilon > 0$ which satisfies (3.8), there exists a $T_3' > T_2$ such that

$$x(t) > \frac{r_1 - \dfrac{q}{m_1}}{b_1 + \dfrac{a_1}{k_1}} - \varepsilon \overset{\text{def}}{=} m_1^{(1)}, \quad \text{for all } t > T_3'. \tag{3.19}$$

From the second equation of system (1.1), we have

$$\dot{y} \geq y\Big(r_2 - b_2 y - \frac{a_2 y}{k_2}\Big). \tag{3.20}$$

Applying Lemma 2.1 to (3.20) leads to

$$\liminf_{t\to+\infty} y(t) \geq \frac{r_2}{b_2 + \dfrac{a_2}{k_2}}. \tag{3.21}$$

Hence, for $\varepsilon > 0$ which satisfies (3.8), there exists a $T_3 > T_3'$ such that

$$y(t) > \frac{r_2}{b_2 + \dfrac{a_2}{k_2}} - \varepsilon \overset{\text{def}}{=} m_2^{(1)} \quad \text{for all } t > T_3. \tag{3.22}$$

(3.19) and (3.22) together with the first equation of system (1.1) imply that

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - \frac{Eqx}{m_1 E + m_2 x} \\ &\geq x\Big(r_1 - \frac{Eq}{m_1 E + m_2 m_1^{(1)}} - b_1 x - \frac{a_1 x}{m_2^{(1)} + k_1}\Big) \quad \text{for all } t > T_3. \end{aligned} \tag{3.23}$$

Applying Lemma 2.1 to (3.23) leads to

$$\liminf_{t\to+\infty} x(t) \geq \frac{r_1 - \dfrac{Eq}{m_1 E + m_2 m_1^{(1)}}}{b_1 + \dfrac{a_1}{m_2^{(1)} + k_1}}. \tag{3.24}$$

That is, for $\varepsilon > 0$ which satisfies (3.8), there exists a $T_4' > T_3$ such that

$$x(t) > \frac{r_1 - \dfrac{Eq}{m_1 E + m_2 m_1^{(1)}}}{b_1 + \dfrac{a_1}{m_2^{(1)} + k_1}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_1^{(2)} > 0, \quad \text{for all } t > T_4'. \tag{3.25}$$

From the second equation of system (1.1), we have

$$\dot{y} \geq y\Big(r_2 - b_2 y - \frac{a_2 y}{m_1^{(1)} + k_2}\Big). \tag{3.26}$$

Applying Lemma 2.1 to (3.20) leads to

$$\liminf_{t\to+\infty} y(t) \geq \frac{r_2}{b_2+\dfrac{a_2}{k_2+m_1^{(1)}}}. \tag{3.27}$$

Hence, for $\varepsilon>0$ which satisfies (3.8), there exists a $T_4>T_4'$ such that

$$y(t) > \frac{r_2}{b_2+\dfrac{a_2}{m_1^{(1)}+k_2}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{(2)} \quad \text{for all } t>T_4. \tag{3.28}$$

Also, since $m_1^{(1)}>0,\ m_2^{(1)}>0$, it follows that

$$\frac{Eq}{m_1E+m_2m_1^{(1)}} < \frac{q}{m_1}, \quad \frac{a_1}{m_2^{(1)}+k_1} < \frac{a_1}{k_1}, \quad \frac{a_2}{m_1^{(1)}+k_2} < \frac{a_2}{k_2},$$

and so

$$\begin{aligned} m_1^{(2)} &= \frac{r_1-\dfrac{Eq}{m_1E+m_2m_1^{(1)}}}{b_1+\dfrac{a_1}{m_2^{(1)}+k_1}} - \frac{\varepsilon}{2} > \frac{r_1-\dfrac{q}{m_1}}{b_1+\dfrac{a_1}{k_1}} - \varepsilon = m_1^{(1)};\\ m_2^{(2)} &= \frac{r_2}{b_2+\dfrac{a_2}{m_1^{(1)}+k_2}} - \frac{\varepsilon}{2} > \frac{r_2}{b_2+\dfrac{a_2}{k_2}} - \varepsilon = m_2^{(1)}. \end{aligned} \tag{3.29}$$

Repeating the above procedure, we get four sequences $M_i^{(n)}, m_i^{(n)},\ i=1,2,\ n=1,2,\cdots$, such that for $n\geq 2$

$$\begin{aligned} M_1^{(n)} &= \frac{r_1-\dfrac{Eq}{m_1E+m_2M_1^{(n-1)}}}{b_1+\dfrac{a_1}{M_2^{(n-1)}+k_1}} + \frac{\varepsilon}{n}, \quad M_2^{(n)} = \frac{r_2}{b_2+\dfrac{a_2}{M_1^{(n-1)}+k_2}} + \frac{\varepsilon}{n},\\ m_1^{(n)} &= \frac{r_1-\dfrac{Eq}{m_1E+m_2m_1^{(n-1)}}}{b_1+\dfrac{a_1}{m_2^{(n-1)}+k_1}} - \frac{\varepsilon}{n}, \quad m_2^{(n)} = \frac{r_2}{b_2+\dfrac{a_2}{m_1^{(n-1)}+k_2}} - \frac{\varepsilon}{n}. \end{aligned} \tag{3.30}$$

Obviously,

$$m_i^{(n)} < x_i(t) < M_i^{(n)} \quad \text{for all } t\geq T_{2n}, \quad i=1,2.$$

We claim that the sequences $M_i^{(n)},\ i=1,2$ are strictly decreasing, and the sequences $m_i^{(n)},\ i=1,2$ are strictly increasing. To proof this claim, we will carry out by induction. Firstly, from (3.16) and (3.29) we have

$$M_i^{(2)} < M_i^{(1)}, \quad m_i^{(2)} > m_i^{(1)}, \quad i=1,2.$$

Let us assume now that our claim is true for n, that is,

$$M_i^{(n)} < M_i^{(n-1)}, \quad m_i^{(n)} > m_i^{(n-1)}, \quad i = 1, 2. \tag{3.31}$$

Then

$$\begin{aligned} \frac{Eq}{m_1E + m_2M_1^{(n)}} &> \frac{Eq}{m_1E + m_2M_1^{(n-1)}}, \\ b_1 + \frac{a_1}{M_2^{(n)} + k_1} &> b_1 + \frac{a_1}{M_2^{(n-1)} + k_1}, \\ b_2 + \frac{a_2}{M_1^{(n)} + k_2} &> b_2 + \frac{a_2}{M_1^{(n-1)} + k_2}. \end{aligned} \tag{3.32}$$

From (3.32) and the expression of $M_i^{(n)}$, it immediately follows that

$$\begin{aligned} M_1^{(n+1)} &= \frac{r_1 - \dfrac{Eq}{m_1E + m_2M_1^{(n)}}}{b_1 + \dfrac{a_1}{M_2^{(n)} + k_1}} + \frac{\varepsilon}{n+1} \\ &< \frac{r_1 - \dfrac{Eq}{m_1E + m_2M_1^{(n-1)}}}{b_1 + \dfrac{a_1}{M_2^{(n-1)} + k_1}} + \frac{\varepsilon}{n} = M_1^{(n)}; \\ M_2^{(n+1)} &= \frac{r_2}{b_2 + \dfrac{a_2}{M_1^{(n)} + k_2}} + \frac{\varepsilon}{n+1} \\ &< \frac{r_2}{b_2 + \dfrac{a_2}{M_1^{(n-1)} + k_2}} + \frac{\varepsilon}{n} = M_2^{(n)}. \end{aligned} \tag{3.33}$$

Also, it follows that $m_i^{(n)} > m_i^{(n-1)}$, $i = 1, 2$, then

$$\begin{aligned} \frac{Eq}{m_1E + m_2m_1^{(n)}} &< \frac{Eq}{m_1E + m_2m_1^{(n-1)}}, \\ b_1 + \frac{a_1}{m_2^{(n)} + k_1} &< b_1 + \frac{a_1}{m_2^{(n-1)} + k_1}, \\ b_2 + \frac{a_2}{m_1^{(n)} + k_2} &< b_2 + \frac{a_2}{m_1^{(n-1)} + k_2}. \end{aligned} \tag{3.34}$$

From (3.34) and the expression of $m_i^{(n)}$, it immediately follows that

$$m_1^{(n+1)} = \frac{r_1 - \dfrac{Eq}{m_1E + m_2m_1^{(n)}}}{b_1 + \dfrac{a_1}{m_2^{(n)} + k_1}} - \frac{\varepsilon}{n+1} > \frac{r_1 - \dfrac{Eq}{m_1E + m_2m_1^{(n-1)}}}{b_1 + \dfrac{a_1}{m_2^{(n-1)} + k_1}} - \frac{\varepsilon}{n} = m_1^{(n)},$$

$$m_2^{(n+1)}=\frac{r_2}{b_2+\dfrac{a_2}{m_1^{(n)}+k_2}}-\frac{\varepsilon}{n+1}>\frac{r_2}{b_2+\dfrac{a_2}{m_1^{(n-1)}+k_2}}-\frac{\varepsilon}{n}=m_2^{(n)}. \tag{3.35}$$

The above analysis shows that $M_i^{(n)}$, $i=1,2$ are strictly decreasing, and the sequences $m_i^{(n)}$, $i=1,2$ are strictly increasing. Therefore,

$$\lim_{t\to+\infty}M_1^{(n)}=\overline{x},\quad \lim_{t\to+\infty}M_2^{(n)}=\overline{y},\quad \lim_{t\to+\infty}m_1^{(n)}=\underline{x},\quad \lim_{t\to+\infty}m_2^{(n)}=\underline{y}.$$

Letting $n\to+\infty$ in (3.30), we obtain

$$\begin{aligned}
b_1\overline{x}+\frac{a_1\overline{x}}{\overline{y}+k_1}&=r_1-\frac{Eq}{m_1E+m_2\overline{x}},\\
b_2\overline{y}+\frac{a_2\overline{y}}{\overline{x}+k_2}&=r_2;\\
b_1\underline{x}+\frac{a_1\underline{x}}{\underline{y}+k_1}&=r_1-\frac{Eq}{m_1E+m_2\underline{x}},\\
b_2\underline{y}+\frac{a_2\underline{y}}{\underline{x}+k_2}&=r_2.
\end{aligned}\tag{3.36}$$

(3.36) shows that $(\overline{x},\overline{y})$ and $(\underline{x},\underline{y})$ are positive solutions of the equations

$$\begin{aligned}
b_1x+\frac{a_1x}{y+k_1}&=r_1-\frac{Eq}{m_1E+m_2x},\\
b_2y+\frac{a_2y}{x+k_2}&=r_2.
\end{aligned}\tag{3.37}$$

Already, Theorem 3.1 shows that under assumption (3.1), (3.37) has a unique positive solution $E^*(x^*,y^*)$. Hence, we conclude that

$$\overline{x}=\underline{x}=x^*,\quad \overline{y}=\underline{y}=y^*,$$

that is

$$\lim_{t\to+\infty}x(t)=x^*,\quad \lim_{t\to+\infty}y(t)=y^*.$$

Thus, the unique interior equilibrium $E^*(x^*,y^*)$ is globally attractive. This completes the proof of Theorem 3.2.

As a direct corollary of Theorem 3.2, we have:

Theorem 3.3 *Assume that $A_1>0$, where A_1 is defined by* (3.5), *assume further that $r_1>q/m_1$, then system* (1.1) *admits a unique positive equilibrium $E^*(x^*,y^*)$, which is globally attractive.*

Proof Noting that under the assumption $A_1>0$ and $r_1>q/m_1$, system (3.37) admits a unique positive solution $E^*(x^*,y^*)$. The rest of the proof is the same as that of Theorem 3.2, and we omit the detail here.

4 Numeric Simulations

Example 4.1 Consider the following system

$$\begin{aligned}\dot{x} &= x\Big(1 - x - \frac{2x}{y+1}\Big) - \frac{2x}{0.5+0.5x},\\ \dot{y} &= y\Big(1 - y - \frac{y}{x+1}\Big).\end{aligned} \tag{4.1}$$

Here, corresponding to system (1.1), we take $r_1 = b_1 = E = r_2 = b_2 = a_2 = k_1 = k_2 = 1$, $m_1 = m_2 = 0.5$, $q = 2$, $a_1 = 2$. In this case, by simple computation, one could easily see that

$$1 = r_1 < \frac{qE}{m_1 E + m_2 \frac{r_1}{b_1}} = 2 \tag{4.2}$$

holds, that is, condition (2.13) in Corollary 2.1 holds, and so, it follows from Corollary 2.1 that the boundary equilibrium $(0, 0.5)$ of the system is globally stable. Numeric simulation (Figure 1) supports this assertion.

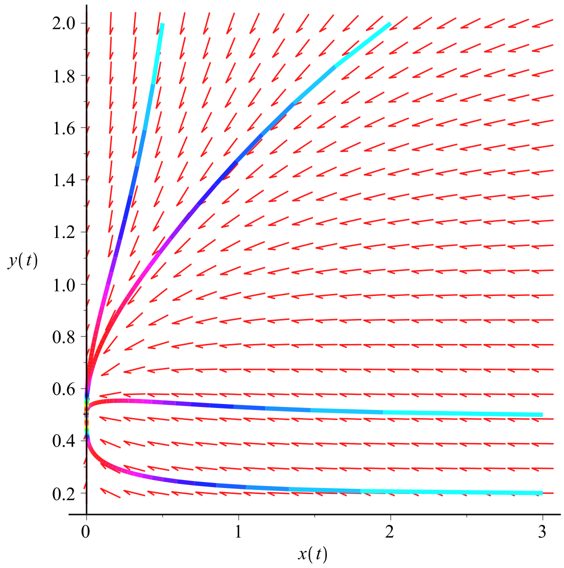

Figure 1: Dynamic behaviors of system (4.1), the initial condition $(x(0), y(0)) = (3, 0.5)$, $(2, 2)$, $(0.5, 2)$ and $(3, 0.2)$, respectively.

Example 4.2 Consider the following system

$$\begin{aligned}\dot{x} &= x\Big(1 - x - \frac{x}{y+1}\Big) - \frac{x}{4+0.5x},\\ \dot{y} &= y\Big(1 - y - \frac{y}{x+4}\Big).\end{aligned} \tag{4.3}$$

Here, corresponding to system (1.1), we take $r_1 = b_1 = a_1 = q = E = r_2 = b_2 = a_2 = k_1 = 1$, $k_2 = 4$, $m_1 = 4$, $m_2 = 0.5$. In this case, by simple computation, one could easily see that

$$b_1 k_2 = 4 > 1 = r_1 > \frac{q}{m_1} = 0.25 \tag{4.4}$$

holds, that is, condition (3.1) in Theorem 3.2 holds, and so, it follows from Theorem 3.2 that the unique positive equilibrium $(0.493\,215\,759\,3, 0.817\,957\,268\,8)$ of the system is globally stable. Numeric simulation (Figure 2) supports this assertion.

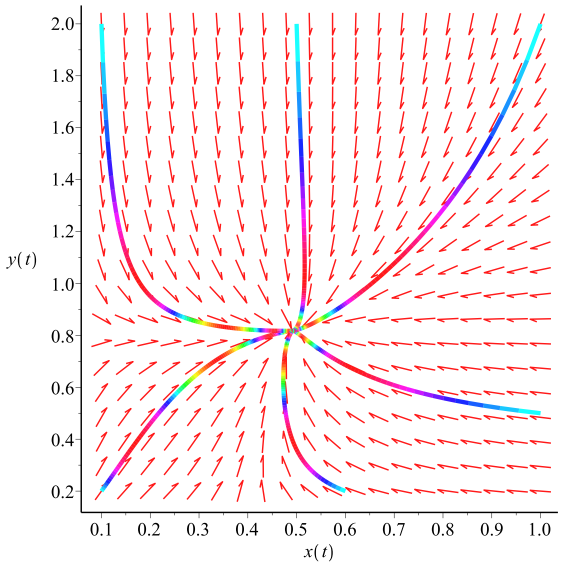

Figure 2: Dynamic behaviors of system (4.3), the initial condition $(x(0), y(0)) = (3, 0.5)$, $(2, 2)$, $(0.5, 2)$ and $(3, 0.2)$, respectively.

5 Discussion

Though there are many works on cooperative system ([1-30]), only recently did scholars ([2,3,25–27,29]) began to study the cooperative or commensal system incorporating harvesting, however, as was shown in the introduction section, both Xie, Chen and Xue [2] and Lei [27] were consider the linear harvesting, and still no scholars investigate the cooperative system with nonlinear harvesting, this motivated us to propose system (1.1).

We first show that overfishing may lead to the extinction of the species. Such kind of property is reflect by the catchablity coefficient q and constant m_1, if q is too large, while m_1 is limited, then inequality (2.1) always holds, consequently, the first species will be driven to extinction.

Our next result is concerned with the global attractivity of the positive equilibrium. Condition (3.1) shows that if b_1k_2 larger then the intrinsic birth rate of the first species, also, if the harvesting is limited, such that $r_1 > q/m_1$ holds, then the two species could be coexist in a stable state. Generally speaking, to overcome the influence of harvesting, increasing the intrinsic rate is a necessary countermeasure. However, our study shows that it also need to restrict the intrinsic rate to satisfy the inequality $r_1 < b_1k_2$. What would happen if the inequality $r_1 < b_1k_2$ does not satisfied, at present we have no answer to this problem, we will leave this for future study.

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