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A Dual Approach for Solving Nonlinear Infinity-Norm Minimization Problems with Applications in Separable Cases

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Abstract

In this paper, we consider nonlinear infinity-norm minimization problems. We device a reliable Lagrangian dual approach for solving this kind of problems and based on this method we propose an algorithm for the mixed linear and nonlinear infinity-norm minimization problems. Numerical results are presented.

Keywords: Infinity-norm minimization problem; Lagrangian dual; linear program.
Mathematics subject classification: 90C30

1. Introduction

In this paper we focus on solving nonlinear infinity-norm minimization problems with applications in separable cases. Consider the following problem, which is to minimize a model defined by nonlinear functions that can depend on multiple parameters:

$$\min_{y\in R^n} \|F(y)\|_\infty, \tag{1.1}$$

where $F : R^n \to R^m$ $(m > n)$, $y \in R^n$ is a vector of variables, and $\|x\|_\infty = \max_{1\le i\le n} |x_i|$.

Generally, problem (1.1) is difficult to solve because of both the nonlinearity of F and the nondifferentiality of the infinity-norm. The usual approach [1] to deal with (1.1) is to use a p-norm to approximate the infinite norm, thanks to the equality $\|F\|_\infty = \lim_{p\to\infty} \|F\|_p$, where $\|x\|_p = (\sum_{i=1}^n |x|_i^p)^{\frac{1}{p}}$, $x \in R^n$.

In this article, we will give a dual approach to handle this problem. As a special case of (1.1), we consider a model of a linear combination of nonlinear functions that can depend on multiple parameters:

$$\min_{x\in R^{n_1},\ y\in R^{n_2}} \|A(y)x - b(y)\|_\infty, \tag{1.2}$$

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where $A(y) \in R^{m \times n_1}$ and $b(y) \in R^m$ (generally $m > n_1 + n_2$) are nonlinear.

This type of problems is very common and has a wide range of applications in different areas, such as inverse problems, signal analysis, mechanical systems, neural networks, communications, robotics and vision, electrical engineering, environmental sciences, to name just a few [2].

Golub and Pereyra [2, 3] have proposed an algorithm for solving the least squares problem:

$$\min_{x \in R^{n_1},\ y \in R^{n_2}} \|A(y)x - b(y)\|_2, \tag{1.3}$$

where A is a variable matrix and b is a variable vector. Obviously, problem (1.1) is more difficult than problem (1.3).

In this paper, we study problem (1.1) and propose an algorithm for finding an optimal solution.

Our paper is organized as follows. In the next section, we present a dual approach for solving the general case of our problem. In Section 3, an algorithm for finding a solution of a special case of problem (1.1) is presented. Numerical results and discussion are reported in Section 4. Conclusions are presented in the last section.

2. A dual approach

Consider problem (1.1)

$$\min_{y \in R^n} \|F(y)\|_\infty \tag{2.1}$$

which can be written as:

$$\begin{aligned} \min \quad & z & (2.2)\\ s.t. \quad & z \geq |F_i(y)|,\ i = 1, \cdots, m. & (2.3) \end{aligned}$$

This is equivalent to the problem:

$$\begin{aligned} \min \quad & z & (2.4)\\ s.t. \quad & z^2 \geq F_i(y)^2,\ i = 1, \cdots, m. & (2.5) \end{aligned}$$

It is also equivalent to

$$\begin{aligned} \min \quad & z^2 & (2.6)\\ s.t. \quad & z^2 \geq F_i(y)^2,\ i = 1, \cdots, m, & (2.7) \end{aligned}$$

because of the following proposition.

Proposition 2.1. *Problem* $\min_{x \in C} f(x)$ *is equivalent to* $\min_{x \in C} g(f(x))$ *for any monotone function* $g : S \to R$, *where* $S = \{f(x) : x \in C\}$.