East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.070816.131016a

Vol. 7, No. 1, pp. 1-20
February 2017

Implicitly Restarted Refined Partially Orthogonal Projection Method with Deflation

Wei Wei and Hua Dai*

Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China.

Received 7 August 2016; Accepted (in revised version) 13 October 2016.

Abstract. In this paper we consider the computation of some eigenpairs with smallest eigenvalues in modulus of large-scale polynomial eigenvalue problem. Recently, a partially orthogonal projection method and its refinement scheme were presented for solving the polynomial eigenvalue problem. The methods preserve the structures and properties of the original polynomial eigenvalue problem. Implicitly updating the starting vector and constructing better projection subspace, we develop an implicitly restarted version of the partially orthogonal projection method. Combining the implicit restarting strategy with the refinement scheme, we present an implicitly restarted refined partially orthogonal projection method. In order to avoid the situation that the converged eigenvalues converge repeatedly in the later iterations, we propose a novel explicit non-equivalence low-rank deflation technique. Finally some numerical experiments show that the implicitly restarted refined partially orthogonal projection method with the explicit non-equivalence low-rank deflation technique is efficient and robust.

AMS subject classifications: 65F15

Key words: Polynomial eigenvalue problem, partially orthogonal projection method, refinement, implicitly restarting, non-equivalence low-rank deflation.

1. Introduction

We consider the polynomial eigenvalue problem of finding a scalar $\lambda \in \mathbf{C}$ and nontrivial vectors $x, y \in \mathbf{C}^n$ such that

$$P(\lambda)x = 0, \quad y^H P(\lambda) = 0, \tag{1.1}$$

where $P(\lambda) = \lambda^d A_d + \lambda^{d-1} A_{d-1} + \cdots + \lambda A_1 + A_0$ with the coefficient matrices $A_i (0 \le i \le d)$ being $n \times n$ large and sparse. The scalar λ and the associated nonzero vectors x and y are called eigenvalue, right and left eigenvectors of the polynomial eigenvalue problem (1.1), respectively. Together, (λ, x) or (λ, x, y) is called an eigenpair of the polynomial eigenvalue problem (1.1). The problem is very general and includes the standard eigenvalue problem

*Corresponding author. *Email addresses:* w.wei@nuaa.edu.cn (W. Wei), hdai@nuaa.edu.cn (H. Dai)

$P(\lambda) = \lambda I - A$, the generalized eigenvalue problem $P(\lambda) = \lambda A - B$, the quadratic eigenvalue problem $P(\lambda) = \lambda^2 A + \lambda B + C$ (see, e.g., [28]) and the cubic eigenvalue problem $P(\lambda) = \lambda^3 A_3 + \lambda^2 A_2 + \lambda A_1 + A_0$ (see, e.g., [16]).

The polynomial eigenvalue problem arises from a remarkable variety of applications, such as vibration analysis of viscoelastic systems [1], structural dynamic analysis [9], stability analysis of control systems [12], numerical simulation of quantum dots [17] and so on. Considerable efforts have been devoted to the polynomial eigenvalue problem in the literature. Gohberg et al. [8] established the mathematical theory concerning matrix polynomials. Gohberg et al. [7], Higham and Tisseur et al. [5, 13], and Chu [4] developed the perturbation theory for the polynomial eigenvalue problem. Tisseur et al. [10, 27], Lawrence and Corless [20] analyzed backward error of the polynomial eigenvalue problem.

In this paper, we consider the computation of some eigenpairs with smallest eigenvalues in modulus of the polynomial eigenvalue problem (1.1). If the coefficient matrix A_0 is singular, then 0 is an eigenvalue of the polynomial eigenvalue problem (1.1), and therefore we assume that A_0 is nonsingular or, equivalently, 0 is not an eigenvalue of the polynomial eigenvalue problem (1.1). If some largest magnitude eigenvalues of the polynomial eigenvalue problem (1.1) are desired, we need only to invert the order of the coefficient matrices $A_i (0 \le i \le d)$ in $P(\lambda)$.

The classical approach for solving the polynomial eigenvalue problem is linearizing the problem (1.1) to produce an equivalent larger generalized eigenvalue problem (see, e.g., [8, 11, 21, 22]), solved using any appropriate eigensolver. The way of linearization is not unique. Using the second companion form of linearization [8], we may convert the polynomial eigenvalue problem (1.1) into the following generalized eigenvalue problem

$$Cy = \theta G y, \tag{1.2}$$

where $\theta = 1/\lambda$,

$$C = \begin{pmatrix} -A_1 & I & & \\ -A_2 & & \ddots & \\ \vdots & & & I \\ -A_d & & & 0 \end{pmatrix}, \qquad G = \begin{pmatrix} A_0 & & & \\ & I & & \\ & & \ddots & \\ & & & I \end{pmatrix},$$

$$y = \begin{pmatrix} x \\ -\lambda A_2 x - \cdots - \lambda^{d-1} A_d x \\ \vdots \\ -\lambda A_d x \end{pmatrix}. \tag{1.3}$$

Since A_0 is nonsingular, the generalized eigenvalue problem (1.3) may be further reduced to the following standard eigenvalue problem

$$\left(G^{-1}C\right) y = \theta y, \tag{1.4}$$