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A Robust Spectral Method for Pricing of American Put Options on Zero-Coupon Bonds

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Abstract. American put options on a zero-coupon bond problem is reformulated as a linear complementarity problem of the option value and approximated by a nonlinear partial differential equation. The equation is solved by an exponential time differencing method combined with a barycentric Legendre interpolation and the Krylov projection algorithm. Numerical examples shows the stability and good accuracy of the method.

AMS subject classifications: 39A05, 65M06, 65M12, 91G60

Key words: Interest rate model, American put bond options, zero-coupon bond, barycentric Legendre method, Greeks.

1. Introduction

A bond is a financial instrument which allows an investor to loan money to an entity (a corporate or governmental) that borrows the funds for a period of time at a fixed interest rate (the coupon) and agrees to pay a fixed amount (the principal) to the investor at maturity. A zero-coupon bond is a bond that makes no periodic interest payments. A bond option is a financial contract which gives the holder the right but not the obligation to buy or sell a bond at a certain price on or before the option expiry date. A European bond option is an option to buy or sell a bond at maturity for a fixed price. On the other hand, an American bond option offers a possibility to buy or sell a bond for a predetermined price on or before the maturity date. A buyer of a bond call option is expecting a decline in interest rates and an increase in bond prices, while a buyer of a put bond option is expecting an increase in interest rates and a decrease in bond prices.

Pricing interest rate contingent claims is a popular field of research in finance [20]. For European zero-coupon bonds the prices of these claims can be calculated as expected terminal payoffs discounted by using the path of the instantaneous risk-free rate r under a risk-adjusted or equivalent martingale measure [12, 25]. American options can be exercised at any time prior to the expiration date, which creates uncertainty and leads to

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substantial difficulties in analytical description of this problem. Nevertheless, the information required can be obtained from an optimal exercise curve related to a free boundary problem. Therefore, numerical or semi-analytical approximation methods have to be involved in the pricing of American style contracts. The valuation of bonds with embedded options have been studied by projective successive over relaxed (PSOR) methods [28], the lattice method [4], explicit finite difference methods [6]. However, in spite of easy implementation, these methods have a slow convergence rate.

In this paper, a barycentric Legendre spectral collocation method is used for numerical valuation of American bond pricing options. Penalising the partial differential complementary problem of American bond options similarly to Ref. [2], we obtain a nonlinear partial differential equation (PDE) and derive its approximate solution in a fixed domain by the collocation scheme mentioned. To discretise the PDE in time, we employ the exponential time differencing method (ETD) studied by Cox and Matthews [8]. A modification of this scheme has been proposed by Kassman and Trefethen [13]. The class of explicit multistep exponential and explicit exponential Runge-Kutta methods have been discussed by Hochbruck and Ostermann [11]. Tangman *et al.* [22] used ETD methods to study European barrier and butterfly spread options for the Black Scholes model and Merton's jump-diffusion model. American style barrier options have been discussed by Gondal [9] and Rambeerich *et al.* [17], who compared the exponential integrators with traditional Crank-Nicolson methods. Here we construct a Krylov subspace by the Arnoldi shift-and-invert method [5] and use it in the evaluation of exponential integrators.

This paper is organized as follows. In Section 2, mathematical models of zero-coupon bonds are presented and American put options on a bond for a short rate model is described. Section 3 deals with a spectral collocation method for these models. In Section 4, we discretise the corresponding systems by exponential time integrators. Numerical results are reported in Section 5, and concluding remarks are in Section 6.

2. Pricing Models of a Zero-Coupon Bond and Options

Let us consider the pricing of zero-coupon bonds and European bond options in the case where a short term risk-free interest rate r is described by the generalized Chan-Karolyi-Longstaff-Schwartz model

$$dr(t) = \kappa\big(\theta - r(t)\big)dt + \sigma r(t)^{\gamma} dW(t), \tag{2.1}$$

where $W(t)$ is a Wiener process, κ the mean-reversion speed, θ the long-term interest rate, σ the volatility, and γ a parameter used for the nesting of term structure models within the framework of the stochastic differential equation (2.1). The pricing equations for zero-coupon bonds and European options on the bonds can be established by traditional no-arbitrage arguments. The pricing equation for both financial products is the same but the boundary conditions differ — cf. [18]. If $\tau^* = T^* - t \in [0, T^*]$ denotes the time of the bond expiration, then the partial differential equation of the bond price $B(r, \tau^*)$ with the