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Fast Algorithms for the Anisotropic LLT Model in Image Denoising

Zhi-Feng Pang[1,2], Li-Lian Wang[*,2] and Yu-Fei Yang[3]

[1] *College of Mathematics and Information Science, Henan University, Kaifeng, 475004, China.*
[2] *Division of Mathematical Sciences, School of Physical and Mathematical Sciences, Nanyang Technological University, 637371, Singapore.*
[3] *College of Mathematics and Econometrics, Hunan University, Changsha, 410082, China.*

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Abstract. In this paper, we propose a new projection method for solving a general minimization problems with two L^1-regularization terms for image denoising. It is related to the split Bregman method, but it avoids solving PDEs in the iteration. We employ the fast iterative shrinkage-thresholding algorithm (FISTA) to speed up the proposed method to a convergence rate $O(k^{-2})$. We also show the convergence of the algorithms. Finally, we apply the methods to the anisotropic Lysaker, Lundervold and Tai (LLT) model and demonstrate their efficiency.

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Key words: Image denoising, anisotropic LLT model, Douglas-Rachford splitting method, split Bregman method, projection method, fast projection method.

1. Introduction

Image denoising is a fundamental task in image processing, which aims to recover a noise-free image u from a noise polluted image f. In general, it can be modeled by $f = u + \eta$, where η is the unknown noise component. Among various methods for finding such a decomposition, the variational approach is to restore u by solving the minimization problem (see, e.g., [2, 14]):

$$\min_{u \in X} \left\{ \mathscr{R}(u) + \lambda \mathscr{F}(u - f) \right\}, \tag{1.1}$$

where the functionals $\mathscr{F}(\cdot)$ and $\mathscr{R}(\cdot)$ are respectively the data fidelity and regularization terms defined on a suitable functional space X, and $\lambda > 0$ is a parameter to balance two

*Corresponding author. *Email addresses:* zhifengpang@163.com (Z.-F. Pang), lilian@ntu.edu.sg (L.-L. Wang), yfyang@hnu.edu.cn (Y.-F. Yang)

terms. The popular total variation (TV) regularized L^2-model, known as the Rudin-Osher-Fatemi (ROF) model [35], takes the form

$$\min_{u\in BV(\Omega)}\Big\{\int_\Omega |Du| + \frac{\lambda}{2}\int_\Omega |u-f|^2\,d\Omega\Big\}, \tag{1.2}$$

where Ω is a bounded domain in R^2 with the Lipschitz boundary, $BV(\Omega)$ is the space of functions with bounded variation, and $\int_\Omega |Du|$ is the total variation of u (see, e.g., [1]). An important variant of the ROF model is as follows (see, e.g., [22, 24, 26]):

$$\min_u\Big\{\int_\Omega \big(|u_x|+|u_y|\big)\,d\Omega + \frac{\lambda}{2}\int_\Omega |u-f|^2\,d\Omega\Big\}. \tag{1.3}$$

The models (1.2) and (1.3) are known as the isotropic and anisotropic ROF model, respectively. They function well for noise removal, and simultaneously preserve discontinuities and edges, so they have been extensively used for a variety of image restoration problems (see, e.g., [14, 33]).

However, as pointed out in [5, 11], the ROF model induces the so-called "staircase effect", as its cost functional is borderline convex (with a linear growth with respect to the image gradients), and it oftentimes produces piecewise constant artificial architects. Some models using higher-order derivative information have been proposed to overcome this drawback (see, e.g., [13, 27, 37, 46]). For instance, Lysaker, Lundervold and Tai [27] suggested the model (termed as the anisotropic LLT model):

$$\min_u\Big\{F(u) := \int_\Omega \big(|u_{xx}|+|u_{yy}|\big)\,d\Omega + \frac{\beta}{2}\int_\Omega |u-f|^2\,d\Omega\Big\}, \quad \beta>0. \tag{1.4}$$

The use of second-order derivatives damps oscillations faster than the total variation regularized model, so (1.4) can reduce the "staircase effect", and produce better approximation to the natural image [27, 46].

Over the past decade, many methods have been developed for the ROF model (1.1). These algorithms typically include (i) the primal approaches, such as artificial time marching algorithms [26, 27, 35], fixed point iterative algorithm [41], and the multigrid method [16]; (ii) the dual methods [9, 10, 12, 15, 30, 32, 38], and (iii) the primal-dual approaches [6, 20, 42], the augmented Lagrangian method [40, 44], and the split Bregman type methods [8, 21, 29, 34, 39, 45]. Moreover, fast graph-cut algorithms [9, 18] have been developed for (1.3).

However, to the best of our knowledge, there are very few discussions on efficient minimization of the anisotropic LLT model (1.4). In this paper, we shall put this model in a more general setting and develop fast algorithms for the minimization problem:

$$\min_u\Big\{E(u) := \int_\Omega \big(|\Lambda_1 u|+|\Lambda_2 u|\big)\,d\Omega + \frac{\gamma}{2}\int_\Omega |u-f|^2 d\Omega\Big\}, \tag{1.5}$$

where Λ_1 and Λ_2 are two bounded linear operators over the admissible function space, and γ is a positive constant. Motivated by the split Bregman method for the ROF model [22],