Commun. Comput. Phys.
doi: 10.4208/cicp.150115.170415s

Vol. **18**, No. 4, pp. 1147-1180
October 2015

Stability and Conservation Properties of Collocated Constraints in Immersogeometric Fluid-Thin Structure Interaction Analysis

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Received 15 January 2015; Accepted (in revised version) 17 April 2015

Abstract. The purpose of this study is to enhance the stability properties of our recently-developed numerical method [D. Kamensky, M.-C. Hsu, D. Schillinger, J.A. Evans, A. Aggarwal, Y. Bazilevs, M.S. Sacks, T.J.R. Hughes, "An immersogeometric variational framework for fluid-structure interaction: Application to bioprosthetic heart valves", Comput. Methods Appl. Mech. Engrg., 284 (2015) 1005–1053] for immersing spline-based representations of shell structures into unsteady viscous incompressible flows. In the cited work, we formulated the fluid-structure interaction (FSI) problem using an augmented Lagrangian to enforce kinematic constraints. We discretized this Lagrangian as a set of collocated constraints, at quadrature points of the surface integration rule for the immersed interface. Because the density of quadrature points is not controlled relative to the fluid discretization, the resulting semi-discrete problem may be over-constrained. Semi-implicit time integration circumvents this difficulty in the fully-discrete scheme. If this time-stepping algorithm is applied to fluid-structure systems that approach steady solutions, though, we find that spatially-oscillating modes of the Lagrange multiplier field can grow over time. In the present work, we stabilize the semi-implicit integration scheme to prevent potential divergence of the multiplier field as time goes to infinity. This stabilized time integration may also be applied in pseudo-time within each time step, giving rise to a fully implicit solution method. We discuss the theoretical implications of this stabilization scheme for several simplified model problems, then demonstrate its practical efficacy through numerical examples.

AMS subject classifications: 65M60, 65M85, 74F10

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Key words: Immersogeometric analysis, fluid-structure interaction, isogeometric analysis, collocation, stabilized methods, augmented Lagrangian.

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1 Introduction

The problem of thin elastic structures undergoing large, unsteady deformations while immersed in incompressible fluid has become a prominent research topic in the computational fluid-structure interaction (FSI) community. Practical instances of this problem include parachutes [1–8], insect flight [9–11], and the valves that direct blood flow through the heart [12–27]. The last topic — heart valve FSI — is currently very active: many of the cited works were published within the past several years. This intense interest is not surprising in light of the immediate practical benefits to be reaped from an improved understanding of the dynamics of both native and prosthetic heart valves.

Hundreds of thousands of diseased heart valves are surgically repaired or replaced by prostheses every year [28, 29]. The most popular type of prosthetic device is one that mimics native heart valves, consisting of thin, flexible leaflets that are pushed open by flow in one direction and pushed closed by flow in the other direction. These biomimetic devices avoid the blood damage that can be caused by mechanical prostheses consisting of rigid parts [28, 29]. To capture critical properties of soft tissue, the prostheses are often themselves composed of biologically derived materials and are hence called bioprosthetic heart valves. The durability of these valves is limited, however, and they often need to be replaced again after 10–15 years due to degradation following from repeated loading [28–30]. Attempts to design more durable prostheses would benefit from understanding the stresses that drive this degradation process. Computational methods of structural analysis have yielded some insights into the mechanics of prosthetic valves and the organs they are modeled after [31–47], but such approaches typically approximate the effect of the fluid crudely, as a uniform pressure applied to each of the valve leaflets. The goal of studies on computational heart valve FSI, such as those cited earlier, is to account for the effect of the fluid more accurately.

In our earlier work on heart valve FSI, we developed a computational method for fluid-thin structure interaction that was initially described by Kamensky et al. [27]. For reasons explained in the cited reference, we developed a non-boundary-fitted method for FSI, in which a shell structure mesh of the heart valve leaflets moves independently of a fluid background mesh. The method was later used by Hsu et al. [26] in conjunction with an arbitrary Lagrangian-Eulerian (ALE) approach [48–50] as a hybrid FSI method — a special case of the fluid-solid interface-tracking/interface-capturing technique (FSITICT) [51, 52] — to study the effect of arterial compliance on valve dynamics.

The thin shell structure in this work is discretized isogeometrically, using non-uniform rational B-spline (NURBS) basis functions [53] to represent both the geometry and the displacement solution of the structure. Hughes et al. [54] introduced isogeometric analysis