Dynamics of Lump Solutions, Rogue Wave Solutions and Traveling Wave Solutions for a $(3+1)$-Dimensional VC-BKP Equation

Ding Guo, Shou-Fu Tian*, Xiu-Bin Wang and Tian-Tian Zhang*

School of Mathematics, China University of Mining and Technology, Xuzhou 221116, P.R. China.

Abstract. The $(3 + 1)$-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation is studied by using the Hirota bilinear method and the graphical representations of the solutions. Breather, lump and rogue wave solutions are obtained and the influence of the parameter choice is analysed. Dynamical behavior of periodic solutions is visually shown in different planes. The rogue waves are determined and localised in time by a long wave limit of a breather with indefinitely large periods. In three dimensions the breathers have different dynamics in different planes. The traveling wave solutions are constructed by the Bäcklund transformation. The traveling wave method is used in construction of exact bright-dark soliton solutions represented by hyperbolic secant and tangent functions. The corresponding 3*D* figures show various properties of the solutions. The results can be used to demonstrate the interactions of shallow water waves and the ship traffic on the surface.

AMS subject classifications: 35Q51, 35Q53, 35C99, 68W30, 74J35

Key words: Breather wave solutions, rogue wave solutions, lump solutions, traveling wave solutions, bright and dark soliton solutions.

1. Introduction

Nonlinear evolution equations (NLEEs) are involved in complex physical phenomena and are used to model various problems in fluid mechanics, plasma physics, optical fibers and solid state physics [1, 2, 6, 24, 28]. In past decades such equations have been studied by mathematicians and physicists. The NLEEs demonstrate both inelastic interactions and admit localised coherent structures [5, 20, 40, 43]. The nonlinear B-type Kadomtsev-Petviashvili (KP) equation is an important representative of such equations — cf. [15, 22]. It has a variety of analytic solutions, the most crucial of which are the rogue, lump and bright-dark soliton solutions [9, 18, 29].

---

*Corresponding author. *Email address:* TS17080009A3@cumt.edu.cn (D. Guo), sftian@cumt.edu.cn, shoufu2006@126.com (S. F. Tian), ttzhang@cumt.edu.cn (T. T. Zhang)

The $(2+1)$-dimensional KP-type equation has the form

$$(u_t + 6uu_x + u_{xxx})_x + 3u_{yy} = 0,$$

and is used to describe the shallow water waves with the weak influence of surface tension and viscosity [14].

The $(3+1)$-dimensional KP-type equation

$$u_{tx} + \alpha u_{xxxy} + \beta(u_x u_y)_x + \gamma u_{xz} = 0$$

can be viewed as a shifted $(2+1)$-dimensional BKP equation for $z = x$. If $\delta \neq 0$, the BKP equation can be written as a $(3+1)$-dimensional KP-type equation — viz.

$$u_{tx} + \alpha u_{xxxy} + \beta(u_x u_y)_x + \gamma u_{xx} + \delta u_{zz} = 0.$$

Among other phenomena, this equation describes the propagation with a uniform speed of shallow water waves of small amplitudes in water canals of a constant depth [8, 12, 16, 17, 21, 25, 30].

Here we focus on the $(3+1)$-dimensional variable-coefficient B-type Kadomtsev-Petviashvili (vc-BKP) equation

$$(u_x + u_y + u_z)_t + \alpha u_{xxxy} + \beta(u_x u_y)_x + \gamma u_{xx} + \delta u_{zz} = 0, \tag{1.1}$$

where u is a complex function of variables x, y, z, t and $\alpha, \beta, \gamma, \delta$ are real constants. This equation can be used to describe propagation of long waves. It also finds various applications in water percolation. If $\alpha = 1, \beta = \chi, \gamma = \delta = -1$, the Eq. (1.1) can be reduced to the BKP equation

$$(u_x + u_y + u_z)_t + u_{xxxy} + \chi(u_x u_y)_x - (u_{xx} + u_{zz}) = 0. \tag{1.2}$$

For a specific choice of parameters, the conservation laws and the multiple-soliton solutions of the Eq. (1.2) have been studied in [3, 42]. The BKP equation can be also used to model water waves with weakly non-linear restoring forces and frequency dispersion. Nevertheless, to the best of our knowledge, the Bäcklund transformation of the Eq. (1.1), the corresponding traveling wave solutions, breather, rogue and lump waves and bright-dark solutions have not been yet studied and we are going to address these problems here.

The rest of the paper is arranged as follows. Section 2 deals with the bilinear representation and breather wave solutions. In Section 3, lump and rogue wave solutions are derived by symbolic computation [4, 7, 10, 11, 19, 23, 26, 27, 31–39, 41, 44, 45]. The Bäcklund transformation considered in Section 4, is used to determine traveling soliton solutions. Section 5 is devoted to bright-dark soliton solutions. Our conclusions are presented in the last section.

2. Bilinear Formalism and Breather Wave Solutions

2.1. Bilinear formalism

We start with a bilinear formalism for (1.1) established by the Hirota bilinear method and the long wave limit method [13].