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Hopf Bifurcation and Singularity Induced Bifurcation in a Leslie-Gower Predator-Prey System with Nonlinear Harvesting

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Abstract. A modified predator-prey model with a new nonlinear harvesting on predator and gestation delay of prey is studied. It is shown that the stability of interior equilibrium point can switch finite times and Hopf bifurcations occur when delay increases through critical values. The properties of the Hopf bifurcations are investigated by the center manifold theorem. Special attention is paid to singularity-induced bifurcations and their state feedback control. Numerical simulations demonstrate the effectiveness of the theoretical results.

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Key words: Predator-prey, nonlinear harvesting, delay, Hopf bifurcation, singularity-induced bifurcation.

1. Introduction and Preliminaries

The dynamical relationship of predator-prey interaction is one of the most important research topics in mathematical biology [7, 13, 22]. In the 1920s, Vito Volterra and Alfred James Lotka established an ecosystem model for one predator population and one single prey population in a constant and uniform environment. This model, termed as standard Volterra-Lotka predator-prey model, is described by differential equations. It attracted considerable attention in applied mathematics and theoretical ecology. On the other hands, in population dynamics of predator-prey systems, a lot of efforts have been spent on applications of the dynamic theory of nonlinear differential equations [7, 12, 13, 22]. However, it was discovered that within the Volterra-Lotka model, the growth rates of the predator and prey populations are not bounded, which contradicts real-world experience. Therefore, Leslie and Gower [23] proposed the following predator-prey model:

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$$\begin{aligned}\dot{x}(t) &= x(t)(a - x(t) - y(t)),\\ \dot{y}(t) &= y(t)\left(d - k\frac{y(t)}{x(t)}\right),\end{aligned} \tag{1.1}$$

where $x(t)$ and $y(t)$ are, respectively, the densities of prey and predator species, a and d the corresponding maximum growth rates of preys and predators, and the parameter $k > 0$ represents the conversion rate from captured preys into predator births — cf. Refs. [7, 13, 22, 23].

It is worth noting that in order to express the interaction and coexistence of biological populations, time delay is often incorporated in the predator-prey models depending on the past history. Generally speaking, it would induce a wealthier dynamical behavior, such as the loss of stability, oscillatory dynamics, various bifurcations (saddle-node, saddle-node-Hopf, Hopf-Andronov, Bogdanov-Takens), and chaos phenomenon [7, 12, 13, 21, 22, 29, 37, 39, 42–44, 46]. Over the years, various predator-prey models with time delay have been studied [7, 13, 21, 22, 29, 43, 44]. One of such models — viz. predator-prey model with gestation delay is ubiquitous in the real world, which can be considered as inherent in biological populations. Taking into account Refs. [7, 12, 13, 22], we incorporate the gestation delay of prey population into the model (1.1), thus obtaining the following delayed predator-prey model

$$\begin{aligned}\dot{x}(t) &= x(t)(a - x(t-\tau) - y(t)),\\ \dot{y}(t) &= y(t)\left(d - k\frac{y(t)}{x(t)}\right).\end{aligned} \tag{1.2}$$

In order to study the economic return from the harvesting effort of mankind on ecological resources, the following economic equation

$$\text{Net Economic Revenue} = \text{Total Revenue} - \text{Total Cost} \tag{1.3}$$

has been adopted [8, 14]. Using the principle (1.3), we are going to study the economic revenue from the harvesting effort $E(t)$ on the predator population in the delayed predator-prey model (1.2). There are two common harvesting modes for predator-prey models — viz. constant and proportional harvesting. The former means that the harvest rate is constant — i.e. it does not depend on population density, and the latter means that the higher number of preys or predators leads to a higher catch. The proportional harvesting is clear improvement to a constant harvesting. However, the harvest increase in the proportional harvesting can become unprofitable due to the market's oversupply.

Here, we propose a new type of nonlinear harvesting. Suppose that T is the total time that each worker needs for harvesting. It usually includes the time T_1 required for looking of predators and the time T_2 for handling them. We also assume that the amount N of predators caught by each worker is proportional to the searching time T_1 and to predators population density $y(t)$, i.e. $N = T_1 y(t)$. Besides, m is the average time spent by a worker on handling of each predator captured. Therefore, we have $T_2 = mN = mT_1 y(t)$, so that $T = T_1(1 + my(t))$. Hence, the amount of predators caught by a worker per unit time is $N/T = y(t)/(1+my(t))$. This harvesting rate is adopted here and the number of predators