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WEAK AND SMOOTH GLOBAL SOLUTION FOR LANDAU-LIFSHITZ-BLOCH-MAXWELL EQUATION*†

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Abstract

This paper is devoted to investigate the existence and uniqueness of the solution of Landau-Lifshitz-Bloch-Maxwell equation. The Landau-Lifshitz-Bloch-Maxwell equation, which fits well for a wide range of temperature, is used to study the dynamics of magnetization vector in a ferromagnetic body. If the initial data is in (H^1, L^2, L^2), the existence of the global weak solution is established. If the initial data is in (H^{m+1}, H^m, H^m) $(m \geq 1)$, the existence and uniqueness of the global smooth solution are established.

Keywords Landau-Lifshitz-Bloch-Maxwell equation; global solution; paramagnetic-ferromagnetic transition; temperature-dependent magnetic theory; Landau-Lifshitz theory

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1 Introduction

In this paper, we consider the periodic initial value problem of the following equations

$$\frac{\partial Z}{\partial t} = \Delta Z + Z \times (\Delta Z + H) - k(1 + \mu|Z|^2)Z, \tag{1.1}$$

$$\frac{\partial E}{\partial t} + \sigma E = \nabla \times H, \tag{1.2}$$

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$$\frac{\partial H}{\partial t} + \beta \frac{\partial Z}{\partial t} = -\nabla \times E, \tag{1.3}$$

$$\nabla \cdot (H + \beta Z) = 0, \quad \nabla \cdot E = 0, \tag{1.4}$$

$$Z(x+2De_i, t) = Z(x,t), \quad H(x+2De_i, t) = H(x,t), \quad E(x+2De_i, t) = E(x,t), \tag{1.5}$$

with the initial conditions

$$Z(x,0) = Z_0(x), \quad H(x,0) = H_0(x), \quad E(x,0) = E_0(x), \quad x \in \mathbb{R}^d, \tag{1.6}$$

where σ, k, μ, β are positive constants, $Z \in \mathbb{R}^3$ is the spin polarization, $H(x,t) = (H_1, H_2, H_3)$ is the magnetic field, $E(x,t) = (E_1(x,t), E_2(x,t), E_3(x,t))$ is the electric field and $H^e = \Delta Z + H$ is the effective magnetic field. $x \in \Omega \subset \mathbb{R}^d$, $d = 2, 3$, $\Omega = \prod\limits_{j=1}^{d}(-D, D)$, $t > 0$.

Here the operator ∇ is defined as follows:

$$\nabla = \nabla_x = \begin{cases} (\partial_{x_1}, \partial_{x_2}, 0), & d = 2, \quad x = (x_1, x_2) \in \mathbb{R}^2, \\ (\partial_{x_1}, \partial_{x_2}, \partial_{x_3}), & d = 3, \quad x = (x_1, x_2, x_3) \in \mathbb{R}^3. \end{cases}$$

System (1.1)-(1.4) was studied in [4] under the additional assumption that temperature is equal to constant. In [3, 4], Berti et. al. proposed a model for the study of the dynamics of magnetization vector in a ferromagnetic body. This model fits well for a wide range of temperature, and then can be used to establish a link between micromagnetics and the phase transition occurring from paramagnetic to ferromagnetic regimes.

System (1.1)-(1.4) generalizes some classical models for magnetically saturated bodies such as the well known Landau-Lifshitz equation. Landau-Lifshitz equation well describes the magnetization dynamics of ferromagnet at low temperature [22]. The Landau- Lifshitz-Gilbert equation is described as follows

$$M_t = M \times \Delta M - \lambda M \times (M \times \Delta M), \quad M \in \mathbb{S}^2, \tag{1.7}$$

where $M(x,t) = (M_1(x,t), M_2(x,t), M_3(x,t))$ is a magnetization vector, $\lambda > 0$ is a Gilbert constant, "$\times$" denotes the vector outer product. Equation (1.7) has been investigated widely. Many important results have been obtained, see [6, 17, 20, 21] and therein references.

Equation (1.7) with $\lambda = 0$ is so-called Schrödinger map [5]. This Schrödinger map has been widely studied in [1, 2, 5, 7, 8, 19, 21, 24–26] and therein references.

The model of [4] is very close to the Landau-Lifshitz-Bloch (LLB) equation. In order to describe the dynamics of magnetization vector Z in a ferromagnetic body for a wide range of temperature, Garanin et. al. [10–12] derived the Landau-Lifshitz-Bloch (LLB) equation from statistical mechanics with the mean field approximation.