Ann. of Appl. Math.
35:2(2019), 139-144

NOTE ON THE STABILITY PROPERTY OF THE VANISHING EQUILIBRIUM POINT OF AN ECOLOGICAL SYSTEM CONSISTING OF A PREDATOR AND STAGE STRUCTURE PREY*†

Xiaoyan Huang, Fengde Chen‡

(*College of Math. and Computer Science, Fuzhou University,
Fuzhou 350116, Fujian, PR China*)

Abstract

We revisit the stability property of an ecological system consisting of a predator and stage structure prey which was proposed by Raid Kamel Naji and Salam Jasim Majeed. By constructing some suitable Lyapunov function and applying the differential inequality theory, we show that the conditions which ensure the local stability of the vanishing point are enough to ensure its global stability. Our result supplements and complements some known results.

Keywords stage structure; predator-prey; global stability

2010 Mathematics Subject Classification 34C25; 92D25; 34D20; 34D40

1 Introduction

During the last decades, many scholars have investigated the dynamic behaviors of the ecosystem, see [1-18] and the references cited therein. Since many species have several states during their life, many scholars investigated the dynamic behaviors of the stage structured ecosystem, see references [1-16]. Such topics as the extinction, persistent and stability have been extensively investigated.

Recently, Naji and Majeed [15] proposed the following ecological system consisting of a predator and stage structure prey

*The research was supported by the National Natural Science Foundation of China under Grant(11601085) and the Natural Science Foundation of Fujian Province (2017J01400).

†Manuscript received January 24, 2019

‡Corresponding author. E-mail: fdchen@263.net

$$\begin{aligned}\frac{\mathrm{d}x_1}{\mathrm{d}T} &= \alpha x_2\Big(1-\frac{x_2}{k}\Big) - r_1x_1 - \beta x_1 - \eta x_1^2 - \beta_1\frac{x_1x_3}{\gamma_1+x_1},\\ \frac{\mathrm{d}x_2}{\mathrm{d}T} &= \beta x_1 - r_2x_2,\\ \frac{\mathrm{d}x_3}{\mathrm{d}T} &= -rx_3 + c\beta_1\frac{x_1x_3}{\gamma_1+x_1} - \eta_1x_3^2,\end{aligned} \tag{1.1}$$

with the initial conditions $x_1(0)>0,\ x_2(0)>0$ and $x_3(0)>0$. By introducing the following transformation

$$y_1=\frac{c\beta_1}{\alpha\gamma_1}x_1,\quad y_2=\frac{c\beta_1}{\alpha\gamma_1}x_2,\quad y_3=\frac{\beta_1}{\alpha\gamma_1}x_1,\quad t=\alpha T, \tag{1.2}$$

system (1.1) could be changed to

$$\begin{aligned}\frac{\mathrm{d}y_1}{\mathrm{d}t} &= y_2(1-w_1y_2) - w_2y_1 - w_3y_1 - w_4y_1^2 - \frac{y_1y_3}{1+w_5y_1},\\ \frac{\mathrm{d}y_2}{\mathrm{d}t} &= w_3y_1 - w_6y_2,\\ \frac{\mathrm{d}y_3}{\mathrm{d}t} &= y_3\Big(-w_7+\frac{y_1}{1+w_5y_1} - w_8y_3\Big),\end{aligned} \tag{1.3}$$

where $w_i,\ i=1,\cdots,8$, are all positive constants. System (1.3) always admits the vanishing equilibrium point $E_0(0,0,0)$, concerned with the stability property of this equilibrium, the authors obtained the following results.

Theorem A *Assume that $w_3<(w_2+w_3)w_6$, then $E_0(0,0,0)$ is locally asymptotically stable.*

Theorem B *Assume that the vanishing equilibrium point $E_0(0,0,0)$ is locally asymptotically stable, then it is globally asymptotically stable in R_+^3, provided that $w_6>1$.*

Now, an interesting issue is proposed: What would happen if $0<w_6\le 1$?

Let's consider the following example.

Example 1.1 Consider the following system

$$\begin{aligned}\frac{\mathrm{d}y_1}{\mathrm{d}t} &= y_2(1-y_2) - 3y_1 - y_1 - y_1^2 - \frac{y_1y_3}{1+y_1},\\ \frac{\mathrm{d}y_2}{\mathrm{d}t} &= y_1 - \frac{1}{2}y_2,\\ \frac{\mathrm{d}y_3}{\mathrm{d}t} &= y_3\Big(-1+\frac{y_1}{1+y_1} - y_3\Big).\end{aligned} \tag{1.4}$$

Here, we assume that $w_1=1,\ w_2=3,\ w_3=1,\ w_4=w_5=w_7=w_8=1,\ w_6=\frac{1}{2}$. Then $w_6<1$ and

$$w_3<(w_2+w_3)w_6 \tag{1.5}$$