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# DYNAMIC BEHAVIORS OF MAY TYPE COOPERATIVE SYSTEM WITH MICHAELIS-MENTEN TYPE HARVESTING*†

Xiangqin Yu, Fengde Chen‡, Liyun Lai

(*College of Math. and Computer Science, Fuzhou University, 350108 Fuzhou, Fujian, PR China*)

**Abstract**

Traditional May type cooperative model incorporating Michaelis-Menten type harvesting is proposed and studied in this paper. Sufficient conditions which ensure the extinction of the first species and the existence of a unique globally attractive positive equilibrium are obtained, respectively. Numeric simulations are carried out to show the feasibility of the main results.

**Keywords** global attractivity; May type cooperative system; Michaelis-Menten type harvesting; iterative method

**2000 Mathematics Subject Classification** 34D23; 92B05; 34D40

## 1 Introduction

The aim of this paper is to investigate the dynamic behaviors of the following May type cooperative model incorporating Michaelis-Menten type harvesting

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - \frac{Eqx}{m_1 E + m_2 x}, \\ \dot{y} &= y\Big(r_2 - b_2 y - \frac{a_2 y}{x + k_2}\Big), \end{aligned} \tag{1.1}$$

where $x$ and $y$ denote the densities of two populations at time $t$. The parameters $r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q, m_1, m_2$ are all positive constants.

During the last decade, many scholars [1-30] investigated the dynamic behaviors of the cooperative system. Yang, Miao, Chen et al [4], Yang and Li [9], Chen, Chen, Li [10], Chen and Xie [11], Han, Xie and Chen [12], Chen and Xie [13], Han, Chen, Xie et al [14], Chen, Yang, Chen et al [15] studied the influence of feedback controls

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‡Corresponding author. E-mail: fdchen@263.net

on the cooperative system; May [1], Xie, Chen and Xue [2], Chen, Wu and Xie [3], Xie, Chen, Yang et al [6], Yang, Xie and Chen [7], Chen, Xie and Chen [8], Chen, Xue, Lin et al [16], Wu and Lin [20], Li, Chen, Chen et al [23], Lin [25], Deng and Huang [26], Lei [27, 28], Chen [29, 30] studied the stability property of the equilibria of cooperative or commensalism model; Chen, Chen and Li [10], Chen and Xie [11], Han, Xie and Chen [12], Chen and Xie [13], Chen, Yang, Chen et al [15], Yang, Xie, Chen et al [19] investigated the persistent property of the cooperative system; Lin [24], Chen [30], Wu [21] investigated the influence of Allee effect on the cooperative system or commensalism system; Xue, Xie and Chen [5], Yang, Xie and Chen [18], Muhammadhaji and Teng [22] investigated the periodic solution or almost periodic solution of the cooperative system.

However, only recently has it attracted the attention of scholars([2,3,25–27,29]) to investigate the influence of harvesting on the cooperative or commensalism model. Xie, Chen and Xue [2] studied the following cooperative system incorporating linear harvesting to the first species

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - Eqx, \\ \dot{y} &= y\Big(r_2 - b_2 y - \frac{a_2 y}{x + k_2}\Big), \end{aligned} \tag{1.2}$$

where $x$ and $y$ denote the densities of two populations at time $t$. The parameters $r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q$ are all positive constants. They showed that if $r_1 > Eq$ holds, then the unique positive equilibrium $E^*(x^*, y^*)$ of system (1.2) is globally attractive.

Lei [27] studied the dynamic behaviors of the following non-selective harvesting May cooperative system incorporating partial closure for the populations

$$\begin{aligned} \dot{x} &= x\Big(r_1 - b_1 x - \frac{a_1 x}{y + k_1}\Big) - Eq_1 mx, \\ \dot{y} &= y\Big(r_2 - b_2 y - \frac{a_2 y}{x + k_2}\Big) - Eq_2 my, \end{aligned} \tag{1.3}$$

where $x$ and $y$ denote the densities of two populations at time $t$. The parameters $r_1, r_2, a_1, a_2, b_1, b_2, k_1, k_2, E, q_1$ and $q_2$ are all positive constants, $E$ is the combined fishing effort used to harvest and $m\,(0 < m < 1)$ is the fraction of the stock available for harvesting. His study showed that the intrinsic growth rate and the fraction of the stocks for the harvesting plays crucial role on the dynamic behaviors of the system, all of the four equilibria maybe globally attractive under some suitable assumption.

It brings to our attention that in system (1.2) and (1.3), the authors chose the linear harvesting. Suck kind of harvesting embodies several unrealistic features and