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UNSTEADY NATURAL CONVECTIVE BOUNDARY LAYER FLOW AND HEAT TRANSFER OF FRACTIONAL SECOND-GRADE NANOFLUIDS WITH DIFFERENT PARTICLE SHAPES*†

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Abstract

The present study is concerned with unsteady natural convective boundary layer flow and heat transfer of fractional second-grade nanofuids for different particle shapes. Nonlinear boundary layer governing equations are formulated with time fractional derivatives in the momentum equation. The governing boundary layer equations of continuity, momentum and energy are reduced by dimensionless variable. Numerical solutions of the momentum and energy equations are obtained by the finite difference method combined with L1-algorithm. The quantites of physical interest are graphically presented and discussed in details. It is found that particle shape, fractional derivative parameter and the Grashof number have profound influences on the the flow and heat transfer.

Keywords second-grade nanofluid; heat transfer; Caputo derivative; particle shape

2000 Mathematics Subject Classification 34K37

1 Introduction

The study of the non-Newtonian fluids has achieved much attention because of well-established applications in a number of processes which occur in industry such as damping and braking devices, personal protective equipment, machining, rocket propellants. The shear stress and shear rate of the non-Newtonian fluids are connected by a relation in a non-linear manner which is generally more complex compared with Newtonian fluid flows. Many research works have been carried out to explore various non-Newtonian fluid flows. Khan et al. [1] studied the heat and

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mass flux models on three-dimensional flow of Burgers fluid over a stretching surface. The magnetohydrodynamics (MHD) three-dimensional flow of Oldroyd-B nanofluids was discussed by Hayat et al. [2]. Ramzan et al. [3] studied on the effect of thermal diffusion on the boundary layer flow of mixed convective viscoelastic nanofluids in a porous medium. Alshomrani et al. [4] researched the heat and mass transfer characteristics of steady-state flow of three-dimensional Burgers nanofluids on biaxially stretched surfaces. Bilal et al. [5] investigated the three-dimensional radiation flow of Burgers nanofluids with mass flux effect. Effect of nonlinear radiation on the flow of MHD Carreau nanofluids on a tensile surface with zero mass flux was analyzed by Lu et al. [6]. One of the most popular models for non-Newtonian fluids is the second-grade fluid model which was configured firstly by Coleman and Noll [7]. Subsequently, Bose et al. [8] made a study on Couette flow of second-grade fluid through a porous medium with suction. Exact solutions of a second-grade fluid movement owing to cylinder vibration were presented by Vieru et al. [9]. Jamil and Fetecau [10] discussed exact analytic solutions of rotating flows of a second-grade fluid between cylindrical regions.

Fractional derivatives are better than integer order models in some applications because they can describe the hereditary and memory properties of diverse substances. For example, complex kinetics can be accurately described, and it can also effectively treat viscoelastic properties. The study of fractional derivative models of non-Newtonian fluids generally begins with classical differential equations, which generally use fractional operators instead of integer-order time derivative. Natural convection flow of a second-grade fluid with non-integer order time-fractional derivatives was studied by Imran et al. [11]. With the consideration of Soret and Dufour effects, Zhao et al. [12] introduced the fractional derivative to characterise the natural convection heat and mass transfer of a MHD viscoelastic fluid in a porous medium. Ming et al. [13] derived analytical solutions of a class of new multi-term fractional-order partial differential equations. Rasheed et al. [14] discussed an unsteady flow of an anomalous Oldroyd-B fluid for solving fractional equation. Zhao et al. [15] studied unsteady natural convection heat transfer of generalized Oldroyd-B fluid in a porous medium saturated with modified fractional Darcy's law. Smooth travelling wave solutions of two fractional flow equations from porous media resulting were analyzed by Hönig et al. [16].

The second-grade fluid model is one of the non-Newtonian models, but it is rare to analyze its fractional derivative. This work is aimed to study the natural convection flow of an incompressible fractional second-grade nanofluid near a vertical plate with different particle shapes, and introduce Caputo fractional derivatives into the stress tensor component. The expressions for dimensionless velocity and