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Modeling the Transmission of West Nile Virus with *Wolbachia* in a Heterogeneous Environment*

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Abstract *Wolbachia* are maternally transmitted endosymbiotic bacteria. To investigate the effect of *Wolbachia* on the spreading and vanishing of West Nile virus, we construct a reaction-diffusion model associated with the *Wolbachia* parameter in a heterogeneous environment, which has nonlinear infectious disease parameters. Based on the spectral radius of next infection operator and the related eigenvalue problem, we present a corresponding explicit expression describing the basic reproduction number. Furthermore, utilizing this number, we not only give out the stability of disease-free equilibrium, but also analyze the uniqueness and globally asymptotic behavior of endemic equilibrium. Our theoretical results and numerical simulations indicate that only if *Wolbachia* reach a certain magnitude in mosquitoes, it can be effective in the control of West Nile virus.

Keywords West Nile virus model, *Wolbachia*, Basic reproduction number, Stability.

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1. Introduction and Model formulation

With the development of economy, the excessive exploitation and unreasonable use of the nature resources aggravate the environmental problems, and the habitats of animals are suffering more and more destruction [32]. Some viruses, which originally transmitted only from animal to animal, are now found to be showing signs of human-to-animal or human-to-human transmission. In this paper, we will focus on West Nile virus(WNv) whose transmission process accords with the aforementioned animal-to-human mechanism.

The WNv, a member of the Flavivirus, usually transmitted between birds and mosquitoes [17]. A wide range of vertebrate including mankind are likely to be accidental hosts of this virus [2]. When birds are bitten by an infected mosquito, the birds' titers in the body continue to rise for three to five days [9], and then the bird transmit the virus to the mosquitoes that bite them [26]. Although mosquitoes may avoid the disadvantageous effects of WNv, birds (especially corvids) have high mortality risk caused by this virus. In 1937, WNv was isolated and identified in the blood of an Ugandan woman [4] for the first time. To begin with, the virus

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was prevalent throughout Israel, France, South Africa, Algeria, Romania, Czech republic, Congo, Russia, etc. Subsequently, in August 1999, it broke out in North America and spread rapidly across much of the country [2, 4, 8, 16]. Except for causing onset and death of large numbers of birds and other wildlife, WNv has also influenced some human [3]. In 2011, WNv was isolated from mosquito samples in Xinjiang Uygur Autonomous Region of China [12]. A large number of serological studies have proved that there once existed diseases caused by WNv infection in this area. WNv has constantly threatened public health constantly, and impeded the development of economy seriously around the world over the last few decades. Therefore, it is of great practical significance to study the transmission mechanism of this virus and take effective measures to control it.

The mathematical model plays a crucial role in terms of preventing and controlling diseases, which can not only help us understand the transmission mechanism of infectious diseases, but also have an impact on predicting, estimating and guiding its development tendency. In order to control and eliminate WNv, we need to pay close attention to how it sustains between organisms and how to keep the morbidity and mortality within certain levels. Faced with these problems, Wonham *et al.* [31] developed an SIR model of WNv cross-infection involving mosquitoes and birds, as well as provided a simple way to determine the control levels of mosquitoes. In literature [9], the authors incorporated the factor of vertical transmission to a WNv model, and found that if the vertical propagation coefficient was high enough, the endemic proportion of infected birds would rise. In [24], researchers were more interested in identifying predictors of WNv incidence in mosquitoes based on a Bayesian space-time model. According to a single-season model to WNv, authors proved that the strategies to decrease mosquitoes and personal protection may prevent WNv effectively in [6]. Besides, Lin and Zhu *et al.* [13, 20] established a reaction-diffusion system with free boundary to explore the transmission mechanism of time and space based on a WNv model between mosquitoes and birds. To study the transmission rates and traveling waves of WNv, Lewis *et al.* [16, 17] initiated a survey of spatial-temporal transmission of disease and considered the spatially-independent and spatially-dependent WNv models with the following forms respectively,

$$\begin{cases} \frac{dI_b}{dt} = \alpha_b\beta_b\frac{N_b-I_b}{N_b}I_m - \gamma_b I_b, \\ \frac{dI_m}{dt} = \alpha_m\beta_b\frac{A_m-I_m}{N_b}I_b - d_m I_m, \end{cases} \tag{1.1}$$

and

$$\begin{cases} \frac{\partial I_b}{\partial t} = d_1\Delta I_b + \alpha_b\beta_b\frac{N_b-I_b}{N_b}I_m - \gamma_b I_b, & t\in(0,+\infty), x\in\Omega, \\ \frac{\partial I_m}{\partial t} = d_2\Delta I_m + \alpha_m\beta_b\frac{A_m-I_m}{N_b}I_b - d_m I_m, & t\in(0,+\infty), x\in\Omega, \\ I_b(0,x) = I_{b,0}(x),\ I_m(0,x) = I_{m,0}(x), & x\in\overline{\Omega}. \end{cases} \tag{1.2}$$

The parameters and variables mentioned above are defined in Table 1.

Meanwhile, Lewis *et al.* exhibited the corresponding basic reproduction number to problem (1.1) as follows:

$$\Re_0 = \sqrt{\frac{\frac{A_m}{N_b}\alpha_m\alpha_b\beta_b^2}{d_m\gamma_b}}. \tag{1.3}$$