

Qualitative Analysis of a Predator-prey System with Ratio-dependent and Modified Leslie-Gower Functional Response*

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Abstract In this paper, a predator-prey model with ratio-dependent and modified Leslie-Gower functional response subject to homogeneous Neumann boundary condition is considered. First, properties of the constant positive stationary solution are shown, including the existence, nonexistence, multiplicity and stability. In addition, a comparatively characterization of the stability is obtained. Moreover, the existing result of global stability is improved. Finally, properties of nonconstant positive stationary solutions are further studied. By a priori estimate and the theory of Leray-Schauder degree, it is shown that nonconstant positive stationary solutions may exist when the system has two constant positive stationary solutions.

Keywords Predator-prey model, Modified Leslie-Gower functional response, Stability, Nonconstant positive stationary solution.

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1. Introduction

The investigations on the relationship between predator and prey are of fundamental importance in mathematical ecology, the predator-prey models have been extensively studied during recent years. More and more realistic models have been derived by virtue of laboratory experiments and observations since the Lotka-Volterra predator-prey model. In [13], Leslie emphasized that the growth rate of the prey and predator has an upper limit and proposed the following predator-prey model

$$\begin{cases} \frac{du}{dt} = u(a - bu) - p(u)v, & t > 0, \\ \frac{dv}{dt} = v \left(d - \frac{hv}{u} \right), & t > 0, \\ u(0) > 0, \quad v(0) > 0, \end{cases} \quad (1.1)$$

where u and v represent the species densities of prey and predator respectively. The term hv/u is usually called the Leslie-Gower functional response, which explains the loss of predator species due to the rarity of its favorite food u . The term $p(u)$ is the functional response of predator to prey. As $p(u)$ is Holling-II type, (1.1) has

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been shown to exhibit quite rich behaviors, such as stable limit cycles, semi-stable limit cycles, global stability of the unique constant positive stationary solution, bifurcation and periodic solutions, see [5, 6, 8, 9, 18, 19, 25].

Taking into account the inhomogeneous distribution of the species in different spatial locations, and the natural tendency of each species to diffuse to areas of smaller population concentration, (1.1) becomes the following system

$$\begin{cases} \frac{\partial u}{\partial t} - d_1 \Delta u = u(a - bu) - p(u)v, & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} - d_2 \Delta v = v \left(d - \frac{hv}{u} \right), & x \in \Omega, t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x) > 0, v(x, 0) = v_0(x) > 0, & x \in \bar{\Omega}. \end{cases} \quad (1.2)$$

As $p(u) = \beta u$, Du and Hsu [7] obtained the global stability of the unique constant positive stationary solution and conjectured that the global stability is true without any restriction. So nontrivial spatial patterns may not be expected for (1.2) with $p(u) = \beta u$. As $p(u)$ is taken as $u/(m+u)$, Peng and Wang [21, 22] analyzed the global stability of the unique constant stationary solution and deduced that nonconstant positive stationary solutions may exist for (1.2).

The functional response of predator to prey can be classified as prey-dependent and predator-dependent types. The prey-dependent functional response only involves the prey u , which means that the prey density alone determines the predation behavior of the predator. However, some recent numerical examples from biological control reveal that the classical prey-dependent functional response can provide contrast to the realistic observations [24]. Moreover, there is growing biological and physiological evidence that in many cases, especially when predators have to search, share and compete for food, a more suitable functional response should be the so-called ratio-dependent one, which is predator-dependent [1–3]. For the predator-prey model with ratio-dependent functional response, one can refer to [12, 15, 27] and references therein. In particular, Peng and Wang [23] investigated the following predator-prey model with ratio-dependent functional response

$$\begin{cases} \frac{\partial u}{\partial t} - d_1 \Delta u = u(\lambda - u) - \frac{\beta uv}{u + mv}, & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} - d_2 \Delta v = v \left(1 - \frac{v}{u} \right), & x \in \Omega, t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) = u_0(x) > 0, v(x, 0) = v_0(x) > 0, & x \in \bar{\Omega}, \end{cases} \quad (1.3)$$

where λ , m , β , d_1 and d_2 are positive constants. In (1.3), $p(u) = u/(u + mv)$ is just the ratio-dependent functional response, in which the parameters m and β account for the saturation rate and the predation rate of the predator, respectively. The Leslie-Gower functional response v/u is also considered in (1.3), while in the case of severe scarcity, predator can switch to other populations but its growth is limited by the fact that its most favorite food is not available in abundance. This situation can be taken care of by adding a positive constant to the denominator, that is, $v/(u + k)$ [4].

Motivated by the above work, this paper is concerned about the following predator-prey model with ratio-dependent and modified Leslie-Gower functional