Journal of Nonlinear Modeling and Analysis
Volume 1, Number 2, June 2019, 251–260
Website:http://jnma.ca
DOI:10.12150/jnma.2019.251

Solitary Waves for the Generalized Nonautonomous Dual-power Nonlinear Schrödinger Equations with Variable Coefficients*

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Abstract In this paper, we study the solitary waves for the generalized nonautonomous dual-power nonlinear Schrödinger equations (DPNLS) with variable coefficients, which could be used to describe the saturation of the nonlinear refractive index and the solitons in photovoltaic-photorefractive materials such as LiNbO3, as well as many nonlinear optics problems. We generalize an explicit similarity transformation, which maps generalized nonautonomous DPNLS equations into ordinary autonomous DPNLS. Moreover, solitary waves of two concrete equations with space-quadratic potential and optical super-lattice potential are investigated.

Keywords Solitary waves, dual-power law, nonlinear Schrödinger equation, variable coefficients.

MSC(2010) 35A09, 35C07, 35C08.

1. Introduction

Phenomena in nonlinear optics and the Bose-Einstein condensates(BECs) are often described by nonlinear Schrödinger equations(NLS) [4–6, 9–11, 14, 16–18, 20, 23, 28]. For example, the wave phenomena observed in fluid dynamics, plasma and elastic media and optical fibers, etc. When we want to understand the physical mechanism of phenomena, exact solutions for the nonlinear Schrödinger equations have to be explored. Moreover, various types of the NLS with non-Kerr nonlinearities, which contain Kerr law, power law, parabolic law, dual-power law as well as the logarithmic law, and other varying potentials were studied by many researchers in [1–3, 7, 11–13, 15, 19, 22, 24–27, 30].

In this paper, we consider solitary waves of the 1D generalized nonautonomous dual-power nonlinear Schrödinger equations with variable coefficients(DPNLS)

$$\mathrm{i}Q_t + D(x,t)Q_{xx} + (lR_1(x,t)|Q|^n + kR_2(x,t)|Q|^{2n})Q + V(x,t)Q = 0, \tag{1.1}$$

where $Q(x,t)$ is the complex envelope of the propagating beam of the modes, x is the propagation distance, and t is the retarded time, l, n, k are arbitrary constants,

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*Han was supported by the Fundamental Research Funds for the Central Universities (Grant 2018MS054).

$D(x,t)$ is the dispersion coefficient, $R_1(x,t), R_2(x,t)$ are the dual-power nonlinearity coefficients, respectively, and $V(x,t)$ is the external potential. This model contains many special types of the NLS with variable coefficients such as the cubic NLS equation, the cubic-quintic(CQ) NLS model, the generalized NLS model, etc [28]. Let $n = 2, k = 0$, l is still arbitrary constants, (1.1) collapses to the Kerr law nonlinear Schrödinger equations with variable coefficients

$$\mathrm{i}Q_t + D(x,t)Q_{xx} + lR_1(x,t)|Q|^2Q + V(x,t)Q = 0, \tag{1.2}$$

which relates to Eq.(1) of [9]. Let $l = 0$, n, k is still arbitrary constants, (1.1) collapses to the power law nonlinear Schrödinger equations with variable coefficients

$$\mathrm{i}Q_t + D(x,t)Q_{xx} + kR_2(x,t)|Q|^{2n}Q + V(x,t)Q = 0, \tag{1.3}$$

which relates to Eq.(1) of [19] and Eq.(2) of [24]. Let $n = 2$, l, k are still arbitrary constants, (1.1) collapses to the parabolic law or CQ law nonlinear Schrödinger equations with variable coefficients

$$\mathrm{i}Q_t + D(x,t)Q_{xx} + (lR_1(x,t)|Q|^2 + kR_2(x,t)|Q|^4)Q + V(x,t)Q = 0, \tag{1.4}$$

which relates to Eq.(1) of [11] with $\gamma(x,t) = 0$. If $V(x,t) = 0$, $D(x,t)$,$R_1(x,t)$,$R_2(x,t)$ are constant coefficients, (1.1) become an autonomous DPNLS equations, which describes the saturation of the nonlinear refractive index, and also serves as a basic model to describe the solitons in photovoltaic-photorefractive materials such as LiNbO3 [2]. However, if $V(x,t), D(x,t), R_1(x,t), R_2(x,t)$ are the functions of x and t respectively, we can't obtain solutions by using the traveling wave method of [27]. To deal with this problem, we hope to use the similarity transformation to obtain solitary waves of (1.1).

In recent years, the similarity transformation has been used to obtain the solution of the nonlinear Schrödinger equations. In [9, 11], the authors solve the nonlinear Schrödinger equations with variable coefficients by using the similarity transformation. In [6–8], the authors also skillfully obtain solitary wave for the coupled nonlinear Schrödinger equations with variable coefficients by using the similarity transformation. In our paper, we firstly deduce the generalized nonautonomous DPNLS to the usual autonomous DPNLS equation (2.2) and obtain stationary solitary waves of the usual autonomous DPNLS (2.2), then using Galilean invariance, we can obtain moving solitary waves of (2.2). Finally using similarity transformation, we could obtain solitary waves of (1.1), so we only need to study solitary waves of (2.2).

Equation (1) can be viewed as the evolution equation $Q_t = \frac{\delta \mathcal{H}}{\delta(\mathrm{i}Q^*)}$, where $\mathcal{H}$ is the Hamiltonian function

$$\mathcal{H} = -\int_{-\infty}^{+\infty} [Q^* D(x,t)Q_{xx} + lR_1(x,t)|Q|^{n+2} + kR_2(x,t)|Q|^{2n+2} + V(x,t)|Q|^2]dx.$$

Generally speaking, the Hamiltonian $\mathcal{H}$ in our model (1) is not conserved. It will be seen that this situation would be changed by employing a similarity transformation technique, and the Hamiltonian $\mathcal{H}$ can only be conserved under some special cases.

The organization of this paper is as follows: In Section 2, we map the generalized nonautonomous DPNLS into the DPNLS by using the similarity transformation, then through some computation, we get stationary solitary waves of the