Journal of Nonlinear Modeling and Analysis http://jnma.ca; http://jnma.ijournal.cn
Volume 1, Number 1, March 2019, 93–105 DOI:10.12150/jnma.2019.93

The Complete Biorthogonal Expansion Theorem and Its Application to a Class of Rectangular Plate Equations*

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Abstract In this paper, we first establish the separable *Hamiltonian* system of rectangular cantilever thin plate bending problems by choosing proper dual vectors. Then using the characteristics of off-diagonal infinite-dimensional *Hamiltonian* operator matrix, we derive the biorthogonal relationships of the eigenfunction systems and based on it we further obtain the complete biorthogonal expansion theorem. Finally, applying this theorem we obtain the general solutions of rectangular cantilever thin plate bending problems with two opposite edges slidingly supported.

Keywords Rectangular cantilever thin plate, Hamiltonian operator, biorthogonal expansion theorem, general solutions, completeness.

MSC(2010) 46E30, 47A70, 47B39, 74B05.

1. Introduction

Rectangular thin plates are important structural components that are used in various engineering applications such as plates in rigid pavements of highways, bridge and houses decks and traffic zones of airports. Bending analysis of rectangular thin plates is one of the challenging issues in theory and engineering, especially for rectangular cantilever thin plates which is an important structural element. Actually its bending has been one of the most difficult problems in the theory of elastic thin plate due to the complexity in both the governing equation and the boundary conditions.

In this paper, we consider the bending problems of cantilever thin plates in the rectangular region $\Omega = \{(x, y) : 0 \leq x \leq a, 0 \leq y \leq b\}$. The governing equations of the plates are

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0, \tag{1.1}$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0, \tag{1.2}$$

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*The authors were supported by NSF of China (Nos. 11671142 and 11771075) and Science and Technology Commission of Shanghai Municipality (No. 18dz2271000).

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0, \tag{1.3}$$

with

$$M_x = -D(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}), \tag{1.4}$$

$$M_y = -D(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2}), \tag{1.5}$$

$$M_{xy} = -D(1-\nu)\frac{\partial^2 W}{\partial x \partial y}, \tag{1.6}$$

where W is the transverse deflection of plate midplane, D is the flexural rigidity, q is the distributed transverse load, M_x, M_y and M_{xy} are the bending moments and the torsional moment, respectively. The internal forces of the plate are

$$Q_x = -D\frac{\partial(\nabla^2 W)}{\partial x}, \tag{1.7}$$

$$Q_y = -D\frac{\partial(\nabla^2 W)}{\partial y}, \tag{1.8}$$

and

$$V_x = Q_x + \frac{\partial M_{xy}}{\partial y}, \tag{1.9}$$

$$V_y = Q_y + \frac{\partial M_{xy}}{\partial x}, \tag{1.10}$$

where Q_x, Q_y, V_x and V_y are shear forces and total shear forces, respectively.

In reference [6], the basic equations for rectangular thin plate were transferred to a *Hamiltonian* canonical equation and the symplectic superposition method was applied to obtain the exact bending solutions. In the end, two numerical examples were provided to illustrate the accuracy of the proposed method. However, the completeness of the eigenfunction systems of the corresponding *Hamiltonian* operator has not been established. In [1] the authors discussed and obtained the completeness of the eigenfunction systems of the *Hamiltonian* operator stated in the rectangular plates with two opposite edges slidingly supported through the symplectic eigenfunction expansion approach. This approach was originated by W. Zhong and X. Zhong [15] to solve a class of eigenvalue problems of non-self-adjoint operators in mathematical physics and it has been applied to various branches of mechanics and engineering sciences, see [2,3,7,8,13,14,16] among others.

In recent years, Luo etc [9,10] further studied a new systematic methodology for theory of elasticity and found that the symplectic orthogonality relationships could be decomposed into two symmetrical and independent sub-orthogonality relationships for the orthotropic plane elasticity and thin plate theory. And the biorthogonal relationships of elasticity was also extended into three-dimensional couple stress problems (see [11]). The biorthogonal relationships not only includes but also is simpler than the symplectic orthogonality relationship. Therefore, the method of biorthogonal expansion in solving the elastic mechanics equations has obvious advantages in calculation than the symplectic eigenfunction expansion method, which makes the calculation more concise. By utilizing this method Hou etc [5] investigated the eigenfunction systems of the *Hamiltonian* operator for the Mindlin