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The Reflection of Magneto-Thermoelastic P and SV Waves at a Solid Half Space Using Dual-Phase-Lag Model

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Abstract. The dual-phase-lag heat transfer model is employed to study the reflection phenomena of P and SV waves from a surface of a semi-infinite magneto-thermoelastic solid. The ratios of reflection coefficients to that of incident coefficients are obtained for P- and SV-wave cases. The results for partition of the energy for various values of the angle of incidence are computed numerically under the stress-free and rigidly fixed thermally insulated boundaries. The reflection coefficients are depending on the angle of incidence, magnetic field, phase lags and other material constants. Results show that the sum of energy ratios is unity at the interface. The results are discussed and depicted graphically.

AMS subject classifications: 73B30, 35Q99

Key words: Reflection, dual-phase-lag model, thermoelastici waves, partition of energy, magneto-thermoelasticity.

1 Introduction

Since the early 1960's there has been an increased usage of composite materials in variety of commercial, aerospace, and military structural configurations involving extreme temperature environments. Therefore, during the past three decades, wide spread attention has been given to thermoelasticity theories which admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories involve a hyperbolic-type heat equation and are referred to as generalized theories. Various authors have formulated these generalized theories on different grounds. For example Lord and Shulman [6] have developed a theory based on a modified heat conduction law which involves heat

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flux rate. This thermoelastic theory includes the finite velocity of thermal wave by correcting the Fourier thermal conduction law by introducing one relaxation time of thermoelastic process. Green and Lindsay [4] formulated a more rigorous theory by including a temperature rate among the constitutive variables; they are considered the finite velocity of the thermal wave by correcting the energy equation and Duhamel-Neumann relation, by introducing two relaxation times of the thermal process. Green and Naghdi [5] developed a theory describing the behavior of a thermoelastic body. This theory is usually called "without energy dissipation".

Tzou [12,13] proposed the dual-phase-lag **(DPL)** model, which describes the interactions between phonons and electrons on the microscopic level as retarding sources causing a delayed response on the macroscopic scale. For macroscopic formulation, it would be convenient to use the DPL mode to investigate of the micro-structural effect on the behavior of heat transfer. The physical meanings and the applicability of the **DPL** model have been supported by the experimental results [14]. The dual-phase-lag **(DPL)** proposed by Chandrasekharaiah and Tzou [1,14] is such a modification of the classical thermoelastic model in which the Fourier law is replaced by an approximation to a modified Fourier law with two different time translations: a phase-lag of the heat flux τ_q and a phase-lag of the temperature gradient τ_θ. A Taylor series approximation of the modified Fourier law, together with the remaining field equations lead to a complete system of equations describing a dual-phase-lag thermoelastic model. The model transmits thermoelastic disturbance in a wave- like manner if the approximation is linear with respect to τ_q and τ_θ, and $0 \leq \tau_\theta < \tau_q$; or quadratic in τ_q and linear in τ_θ, with $\tau_q > 0$ and $\tau_\theta > 0$. This theory is developed in a rational way to produce a fully consistent theory which is able to incorporate thermal pulse transmission in a very logical manner.

In the last few decades a new domain has been developed which investigates the interactions between strain and electromagnetic fields. This discipline is called magneto-elasticity. A stimulus for its development was the possibility of its applications to geophysical problems, certain topics in optics and acoustics, investigations on damping of acoustic waves in a magnetic field, etc. The theory of magneto-thermoelasticity is concerned with the influence of the magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in recent years, because of its application in various branches of science and technology. With the rapid development of polymer science and plastic industry, as well as the wide use of materials under high temperature in modern technology and application of biology and geology in engineering, the theoretical study and applications in viscoelastic materials has become an important task for solid mechanics.

The subject of wave propagation and their reflection and transmission from interfaces in an elastic medium is of great interest since long. These studies help us manifold e.g., the elastic waves propagating through the Earth (called seismic waves) have to travel through different layers and interfaces. The velocities of these waves are influenced by the properties of the layer through which they travel and whenever these waves come across the discontinuities between different layers, the phenomena