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The Weighted Morrey Boundedness of Multilinear Singular Integral Operators on RD-Spaces

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Dedicated to Prof. Shanzhen Lu with admiration on the occasion of his 80th birthday

Abstract. An RD-space $\mathcal{X}$ is a space of homogeneous type in the sense of Coifman and Weiss, which is equipped with a measure satisfying an additional reverse doubling property. In this paper we study the boundedness of multilinear singular integral operators in weighted Morrey spaces within the framework of RD-spaces.

Key Words: RD-space, multilinear Calderón-Zygmund operator, multiple weight, weighted Morrey space.

AMS Subject Classifications: 43A85, 47H60

1 Introduction

The Calderón-Zygmund singular integral operator is a natural generalization of the Hilbert transform and the Riesz transform on the Euclidean space, and its corresponding theories have been proven to be a powerful tool in many aspects of harmonic analysis and partial differential equations. The exploration of multilinear operators is not only motivated by the generalization of linear operators, but also their natural appearance in analysis. The study on the class of multilinear Calderón-Zygmund operators was preliminarily considered by Coifman and Meyer in [4] and was systematically discussed by Grafakos and Torres in [10]. Since then, a lot of papers focus on this field of research. The following are some recent development: Grafakos et al. [8] provided a self-contained proof of the multilinear extension of the Marcinkiewicz real method interpolation theorem; Mo and Lu [18] established the boundedness of commutators generated by the multilinear Calderón-Zygmund type singular integrals and Lipschitz functions on the

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Triebel-Lizorkin space and Lipschitz spaces; Lin et al. [17] obtained some sharp maximal estimates for multilinear commutators of multilinear strongly singular Calderón-Zygmund operators.

On the other hand, the weighted inequalities arise naturally in Fourier analysis, but their use is best justified by the variety of applications in which they appear; see, for instance, Grafakos [7]. Grafakos and Torres [11] established the weighted estimates with A_p weights for the multilinear Calderón-Zygmund operator and the corresponding maximal operator. The corresponding results are generalized to the multilinear singular integral operators with non-smooth kernels by Hu and Lu [13]. In 2009, Lerner et al. [16] developed the theory of the multiple A_p weight, which was applied to the weighted estimates for multilinear Calderón-Zygmund operators. Recently, Chen et al. [1] established the multiple weighted norm inequalities for the maximal vector-valued multilinear Calderón-Zygmund operators. In 2013, Wang and Yi [22] studied the boundedness properties of multilinear Calderón-Zygmund operators on products of weighted Morrey spaces with multiple weights. We mention that the classical Morrey spaces were introduced by Morrey [19] in order to study the existence and differentiability properties of the solutions of second-order elliptic partial differential equations, and the weighted Morrey space was first introduced by Komori and Shirai [15].

With the deepening of theoretical research, it has been discovered that a number of remarkable problems do not need to be embedded in the framework of Euclidean spaces equipped with Lebesgue measures. A prime example is the spaces of homogeneous type introduced by Coifman and Weiss [5, 6]. In the past decade, the analysis on RD-space, which is a special space of homogeneous type equipped with measures satisfying an additional reverse doubling property, has been greatly developed; see, for instance, [3, 9, 12, 14, 20, 23–25] and the references therein. It should be pointed out that the theory of multilinear operators on RD-spaces has been discussed by Grafakos et al. [9] in a unified way. Moreover, Kokilashvili and Meskhi [14] introduced the weighted Morrey space on RD-spaces.

Inspired by the above works, the main purpose of this paper is to establish the boundedness properties of multilinear Caldeŕon-Zygmund operators on products of weighted Morrey spaces with multiple weights defined on RD-spaces.

To illustrate our main results, we first review some necessary definitions and notation. We begin with the definition of RD-space.

Definition 1.1. *Let $(\mathcal{X}, d)$ be a metric space and for any $x \in \mathcal{X}$ and $r \in (0, \infty)$, let the ball $B(x, r) := \{y \in \mathcal{X} : d(x, y) < r\}$. Suppose that μ is a regular Borel measure defined on a σ-algebra which contains all Borel sets induced by the open balls $\{B(x, r) : x \in \mathcal{X}, r > 0\}$, and that $0 < \mu(B(x, r)) < \infty$ for all $x \in \mathcal{X}$ and $r \in (0, \infty)$.*

(i) The triple $(\mathcal{X}, d, \mu)$ is called a space of homogeneous type if there exists a constant $C_1 \in [1, \infty)$ such that, for any $x \in \mathcal{X}$ and $r \in (0, \infty)$,

$$\mu(B(x, 2r)) \leq C_1 \mu(B(x, r)). \tag{1.1}$$