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SSP IMEX Runge-Kutta WENO Scheme for Generalized Rosenau-KdV-RLW Equation

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Abstract. In this article, we present a third-order weighted essentially non-oscillatory (WENO) method for generalized Rosenau-KdV-RLW equation. The third order finite difference WENO reconstruction and central finite differences are applied to discrete advection terms and other terms, respectively, in spatial discretization. In order to achieve the third order accuracy both in space and time, four stage third-order L-stable SSP Implicit-Explicit Runge-Kutta method (Third-order SSP EXRK method and third-order DIRK method) is applied to temporal discretization. The high order accuracy and essentially non-oscillatory property of finite difference WENO reconstruction are shown for solitary wave and shock wave for Rosenau-KdV and Rosenau-KdV-RLW equations. The efficiency, reliability and excellent SSP property of the numerical scheme are demonstrated by several numerical experiments with large CFL number.

AMS subject classifications: 65M60, 35L65

Key words: Rosenau-KdV-RLW equation, WENO reconstruction, finite difference method, SSP implicit-explicit Runge-Kutta method.

1 Introduction

The nonlinear wave behavior is one of the active scientific research areas during the past several decades. Numerical solution of nonlinear wave equations is significantly necessary since most of these types of equations are not solvable analytically in the case of the nonlinear terms are included.

Many mathematical models, especially nonlinear partial differential equations describe various types of wave behavior in nature. Typically, the KdV equation (Korteweg-de Vries equation) is suitable for small-amplitude long waves on the surface of the subject, such as shallow water waves, ion sound waves, and longitudinal astigmatic waves.

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RLW equation (Regularized Long-Wave equation) can describe not only shallow water waves, but also nonlinear dispersive waves, ion-acoustic plasma waves, magnetohydrodynamic plasma waves. The Rosenau equation [1] was proposed for explaining the dynamic of dense discrete systems since the case of wave-wave and wave-wall interactions can not be explained by the KdV and RLW equations.

In order to further consider the nonlinear wave behavior, the viscous term u_{xxx} or u_{xxt} need to be included in Rosenau equation, which leads to the achievement of Rosenau-RLW equation:

$$u_t+\alpha u_x+\delta u_{xxt}+\nu u_{xxxxt}+\varepsilon(u^p)_x=0. \tag{1.1}$$

or Rosenau-KdV equation:

$$u_t+\nu u_{xxxxt}+\alpha u_x+\theta u_{xxx}+\varepsilon(u^p)_x=0. \tag{1.2}$$

There have been many difficulties in evaluating analytical solutions of nonlinear dispersive wave equations and so on the development of numerical schemes. Even so, one derived the solitary wave solution and singular soliton solution for the Rosenau-KdV equation by the ansatz method as well as the semi-inverse variational principle [2] while the shock wave solution of this equation was determined for two particular values of the power law nonlinearity parameter $p=3$ and $p=5$ by Ebadi [3].

Significant numerical studies have been done on the Rosenau-KdV equation [4, 5]. Two-level nonlinear implicit Crank-Nicolson difference scheme and three-level linear-implicit difference scheme were presented to solve two-dimensional generalized Rosenau-KdV equation by Atouani [4]. Their experiment proved that both schemes were uniquely solvable, unconditionally stable and second-order convergent in L_1 norm, the linearized scheme was more effective in terms of accuracy and computational cost. Wang and Dai [5] proposed a conservative unconditionally stable finite difference scheme with $O(h^4+\tau^2)$ for the generalized Rosenau-KdV equation in both one and two dimension, where h is spatial step and τ is temporal step, respectively.

A mass-preserving scheme which combined a high-order compact scheme and a three-level average difference iterative algorithm was analyzed and tested for the Rosenau-RLW equation in [6]. In their work, they focused on the development of the approach for solving the nonlinear implicit scheme in aim to improve the accuracy of approximate solutions. The Rosenau-RLW equation was also solved by second-order nonlinear finite element Galerkin-Crank-Nicolson method which was linearized by predictor-correction extrapolation technique in [7]. An energy conservative two-level fourth-order nonlinear implicit compact difference scheme for three dimensional Rosenau-RLW equation was designed by Li [8] and an iterative algorithm was introduced to generate this nonlinear algebraical system.

In this paper, we focus on one-dimensional generalized Rosenau-KdV-RLW equation. This model is difficult to solve numerically because of the excessive computational cost caused by high order mixed derivative term and the selective wave behavior caused by the power law nonlinearity term. In order to keep this model in a generalized setting, the