Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **14**, No. 2, pp. 415-441

DOI: 10.4208/aamm.OA-2020-0257
April 2022

Multiderivative Combined Dissipative Compact Scheme Satisfying Geometric Conservation Law III: Characteristic-Wise Hybrid Method

Yi Jiang[1,2,*], Meiliang Mao[2,3], Xiaogang Deng[1] and Huayong Liu[2]

[1] *Institute of System Engineering, Academy of Military Sciences, Beijing 100082, China*
[2] *State Key Laboratory of Aerodynamics, China Aerodynamics Research and Development Center, P.O. Box 211, Mianyang, Sichuan 621000, China*
[3] *Computational Aerodynamics Institute, China Aerodynamics Research and Development Center, P.O. Box 211, Mianyang, Sichuan 621000, China*

Received 16 August 2020; Accepted (in revised version) 21 July 2021

Abstract. Based on newly developed weight-based smoothness detectors and nonlinear interpolations designed to capture discontinuities for the multiderivative combined dissipative compact scheme (MDCS), hybrid linear and nonlinear interpolations are proposed to form hybrid MDCS. These detectors are derived from the weights used for the nonlinear interpolations and can provide suitable switches between the linear and the nonlinear schemes to realize the characteristics for the hybrid MDCS of capturing discontinuities and maintaining high resolution in the region without large discontinuities. To save computational cost, the nonlinear scheme with characteristic decomposition is only applied in the detected discontinuities region by specially designed hybrid strategy. Typical tests show that the hybrid MDCS is capable of capturing discontinuities and maintaining high resolution power for the smooth region at the same time. With the satisfaction of the geometric conservative law (GCL), the MDCS is further applied on curvilinear mesh to present its promising capability of handling pragmatic simulations.

AMS subject classifications: 76Fxx, 76Gxx

Key words: Hybrid multiderivative combined dissipative compact scheme, high resolution power, discontinuities capturing, geometric conservation law, curvilinear mesh complex geometry.

1 Introduction

The multiderivative combined dissipative compact scheme (MDCS) with linear interpolation has been proposed in our previous study [22]. The linear MDCS has high resolution to resolve small-scale flow structures and can satisfy the geometric conservation law

*Corresponding author.
Emails: yijiang@mail.ustc.edu.cn (Y. Jiang), mml219@163.com (M. Mao)

(GCL). It is extremely important for the finite difference scheme to satisfy the GCL in the purpose of enabling pragmatic applications on complex curvilinear meshes [7,9,21,31,33, 34,38,46,47]. Due to the fact that the GCL is satisfied, pragmatic applications have been successfully practiced by the linear MDCS on complex curvilinear meshes [23]. However, the linear MDCS cannot capture discontinuities, because it does not have discontinuities capturing mechanism.

In order to capture discontinuities, many schemes have been proposed, such as total variation diminishing (TVD) schemes [14], essentially non-oscillatory (ENO) [15], and weighted essentially non-oscillatory (WENO) [30] schemes. The TVD schemes capture discontinuities with sufficient numerical dissipation, however, they degenerate to first-order and produce overabundant numerical dissipation near critical points [2,5,41], where zero value of low-order spatial derivatives occurs, even though the solution is still smooth. Compared with the TVD schemes, the ENO schemes reduce numerical dissipation and capture discontinuities by selecting the smoothest stencil in a set of candidate stencils [15,42]. The WENO schemes are constructed by a weighted average of approximations from all candidate stencils. Based on smoothness indicators, the weights are designed to construct the property of capturing discontinuities for the WENO schemes, and an optimal linear scheme can be restored by the WENO schemes in smooth regions [18,30]. Although the WENO schemes have been widely used, excessive dissipation significantly dissipating small-scale flow structures has been observed in applications of these schemes on turbulent flow simulations [50]. Moreover, it is usually hard to recover the optimal linear scheme in non-smooth regions due to the weights depending on local flow feature, and the optimal linear scheme is often more dissipative than low-dissipation compact linear scheme [12].

For the purpose of constructing a scheme with the property of capturing discontinuities and high resolution for the regions without discontinuities, a promising approach is to propose hybrid schemes, which combine a more dissipative nonlinear scheme with relatively low resolution power and a less dissipative linear scheme with relatively high resolution power [17]. The combination of the nonlinear and the linear scheme usually depends on a discontinuity detector [1,25,29,36]. Based on the discontinuity detector, the hybrid schemes provide transitions between linear and nonlinear schemes. The transition can be abrupt [1] or smooth [40]. Since abrupt transition can lead to undesired spurious waves at the interfaces between the smooth regions and the discontinuities [1,27], smooth transition is usually adopted to construct hybrid scheme in recent studies [17,40]. In the present study, an alternative approach to avoid undesired numerical contamination from the transition between the linear and nonlinear schemes is that the transition is chosen some points away from the interface between the smooth regions and the discontinuities.

The hybrid MDCS is proposed to capture discontinuities and maintain high resolution in the region without large discontinuities. First, a nonlinear mechanism should be proposed according to the concept of constructing the weighted compact nonlinear schemes (WCNS) [6]. Then, a nonlinear MDCS can be achieved by replacing the dissipative compact interpolation in the linear MDCS with a newly designed nonlinear inter-