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Linearized Double-Shock Approximate Riemann Solver for Augmented Linear Elastic Solid

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Abstract. In this work, in order to capture discontinuities correctly in linear elastic solid, augmented internal energy is defined according to the first law of thermodynamics and Hooke's law. The non-conservative linear elastic system is then rewritten into a conservative form with the help of an augmented total energy equation. We find that the non-physical oscillations occur to the popular HLL and HLLC approximate Riemann solvers when directly applied to simulate the augmented linear elastic solid. We analyze the intrinsic reason by defining a discrepancy factor which can be used to estimate the difference of the total stress across a contact discontinuity, where it is physically required to be continuous. We discover that non-physical oscillations inevitably appear in the vicinity of the contact discontinuity if this factor is away from zero for an approximate Riemann problem solver. In order to overcome this difficulty, we propose an approximate Riemann solver based on the linearized double-shock technique. Theoretical analysis and numerical results show that in comparison to the HLL and HLLC approximate Riemann solvers, the present linearized double-shock Riemann solver can eliminate the non-physical oscillations effectively.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07
Key words: Linear elastic solid, approximate Riemann solver, discrepancy factor.

1. Introduction

In recent decades, various elastic and plastic models, such as hyper-elastic plastic models and hypo-elastic plastic models, have been developed for simulating mechanical behaviors of solid materials. To better understand the performance of those theoretical models, researchers [9–11,20,21] have put much effort into developing exact solutions concerning various models. Those exact solutions are precious and have played an important role in constructing and verifying numerical solvers in the simulation of

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compressible solids. In this work, we focus on developing numerical method for linear elastic solid and verifying it with exact solutions.

A hyper-elastic plastic model usually satisfies the second law of thermodynamics and the corresponding governing system can be written in a conservative form. Garaizar [11] proposed an exact iterative Riemann problem solver for the isotropic hyper-elastic model. Based on the above work, LeFloch and Olsson [15] presented an approximate Riemann solver, which only utilized features of shock waves. Gavrilyuk *et al.* [12] constructed an approximate Riemann solver for the non-conservative non-linear elastic system. Miller [20] presented an exact iterative Riemann solver for the general hyper-elastic system. Barton *et al.* [1] presented another iterative method for finding the exact solution to the Riemann problem with non-linear elasticity. Trangenstein *et al.* [22] constructed an approximate Riemann solver for considering the interaction of elastic waves at cell boundaries.

Compared with hyper-elastic plastic models, a hypo-elastic plastic model might be inconsistent with thermodynamics strictly and often results in a non-conservation governing system. However, such a model bears the advantages of reproducing experimental data accurately (especially for metal materials), introducing plastic deformation naturally, and dealing with complex multi-dimensional boundary problems easily. For a hypo-elastic plastic model, an equation of state (EOS) or Hooke's law is usually applied in the elastic region, the EOS commonly includes the Murnagham equation of state and the Mie-Gruüneisen equation of state. The former is suitable for simulating the solid state at high temperature and high pressure. Tang *et al.* [21] put forward an exact Riemann solver for the hydro-elastoplastic solid. For the latter, Maire *et al.* [19] proposed a nodal-based Riemann solver in the lagrangian coordinate. Chen *et al.* [2] proposed an approximate iterative solver for elastic-plastic Riemann problems. Cheng and colleges [3, 4] developed a two-rarefaction Riemann solver (TRRSE) and Harten-Lax-van Leer-contact (HLLC) approximate Riemann solvers for elastic waves. In their work [3], they found the popular HLLC approximate Riemann solver suffered numerical oscillations. Later, they proposed a multi-material HLLC with both elastic and plastic (MHLLCEP) approximate Riemann solvers [17] to fix the above difficulty by enforcing the continuity of total stress across the contact discontinuity. Recently, Li *et al.* [16] presented another HLLC-type approximate Riemann solver, where the elastic-plastic shear wave was considered, to overcome the above-mentioned problem.

In practice, for metal materials under not very high temperature and pressure, Hooke's law is more appropriate in reflecting the mechanical behavior of metals. This usually leads to linear elastic modeling to the solid. Wilkins [23] extensively investigated the linear elastic model, in which Hooke's law was applied to model the elastic region, and perfect plasticity was adopted to treat the plastic region with an equation of state. As a result, the governing system is non-conservative in the elastic region, while it is conservative in the plastic region. When there is a shock wave in the elastic region, a non-conservation system might lead to incorrect numerical results as found by Gavrilyuk *et al.* [12]. Barton *et al.* [1] and Trangenstein *et al.* [22] also pointed out that the non-conservative system of an elastic model produced non-physical character-