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An Efficient Threshold Dynamics Method for Topology Optimization for Fluids

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Abstract. We propose an efficient threshold dynamics method for topology optimization for fluids modeled with the Stokes equation. The proposed algorithm is based on minimization of an objective energy function that consists of the dissipation power in the fluid and the perimeter approximated by nonlocal energy, subject to a fluid volume constraint and the incompressibility condition. We show that the minimization problem can be solved with an iterative scheme in which the Stokes equation is approximated by a Brinkman equation. The indicator functions of the fluid-solid regions are then updated according to simple convolutions followed by a thresholding step. We prove mathematically that the iterative algorithm has the total energy decaying property. The proposed algorithm is simple and easy to implement. Extensive numerical experiments in both two and three dimensions show that the proposed iteration algorithm converges in much fewer iterations and is more efficient than many existing methods. In addition, the numerical results show that the algorithm is very robust and insensitive to the initial guess and the parameters in the model.

AMS subject classifications: 35K93, 35K05, 65M12, 35Q35, 49Q10, 65M60, 76S05

Key words: Topology optimization, Stokes flow, threshold dynamics method, mixed finite-element method.

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1 Introduction

Topology optimization has become a significant problem due to its application in many industrial problems such as the optimization of transport vehicles and biomechanical structure. The process of topology optimization allows the introduction of new boundaries as part of the solution and is thus more flexible than shape optimization, which requires that the topology be predetermined. It is since the original work by Bendsøe and Kikuchi [8] on homogenization approach to topology optimization that many methods have been developed. For instance, region-based approaches: density interpolation [7], level-set method approach [5], topological derivatives [27], and phase field method [17]; contour-based approaches: evolutionary structural optimization (ESO) approaches [48] and the bi-directional schemes (BESO) [52]; and several others. Topology optimization was first applied to fluid mechanics by Borrvall and Petersson [9] by adapting the concept of density methods to Stokes flows. In [9], the total domain with fluid-solid regions was treated as a porous medium, the Brinkman flow was introduced to obtain a well-posed problem to minimize the total dissipation power, and the discrete optimization problem was further solved with the method of moving asymptotes (MMA) [37] to obtain the optimal designed regions for fluids and solids. Topology optimization in fluid mechanics has since been extended to the Darcy-Stokes flow model [47], Navier-Stokes flow [39], and non-Newtonian flow [33], and it has also been applied in the design of more complicated fluidic devices [6, 28, 29].

The level set based approach [30] has been very popular in topology optimization. It was applied to fluidic topology optimization where the fluid-solid interface is described by the zero-level contour of a level set function. In [2], the authors proposed a method that combines the sensitivity analysis and the level set method for many problems in topology optimization. It has been subsequently extended to multi-phase structure optimization [3], optimal design [4], multiple loads structural optimization [5, 46], and also fluid problems [22, 39].

To ensure well-posedness and mesh-independent solutions, regularization is usually needed in topology optimization. Many robust regularization approaches were introduced. For example, regularizing the optimization problem by introducing fictitious interface energy [51] or penalizing the original objective function by perimeter control functions based on phase field method [34]. The level set based method can also be applied to solve topology optimization problems (e.g. [2] and the reference therein).

The existing numerical methods for solving the topology optimization for Stokes flow problem are quite mature. However, the efficiency of these methods can still be improved which is what we consider in this paper. When the MMA is applied to the topology optimization for Stokes flow [47], an additional sub-optimization problem needs to be solved and the parameters involved also need to be updated in each iteration. In the level set based methods for topology optimization for fluids [2,22], one needs to do either reinitialization/redistancing or some related techniques. These motivate us to design a new iterative algorithm based on the threshold dynamics approach from [13]. The new algorithm