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ALMOST PERIODIC SOLUTION OF A NONAUTONOMOUS MODIFIED LESLIE-GOWER PREDATOR-PREY MODEL WITH NONMONOTONIC FUNCTIONAL RESPONSE AND A PREY REFUGE*†

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Abstract

A nonautonomous modified Leslie-Gower predator-prey model with nonmonotonic functional response and a prey refuge is proposed and studied in this paper. Sufficient conditions which guarantee the permanence, extinction of the prey species and the global stability of the system are obtained, respectively. Also, by constructing a suitable Lyapunov function, some sufficient conditions are obtained for the existence of a unique globally attractive positive almost periodic solution of this model. Our results indicate that the prey refuge has positive effect on the coexistence of the species. Examples together with their numeric simulation show the feasibility of our main results.

Keywords predator; prey; permanence; global stability

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1 Introduction

The traditional Leslie-Gower predator-prey model, which was proposed by Leslie ([1]), which takes the form:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = (r_1 - a_1 P - b_1 H)H, \quad \frac{\mathrm{d}P}{\mathrm{d}t} = \Big(r_2 - a_2 \frac{P}{H}\Big)P, \tag{1.1}$$

where H and P are the densities of prey species and the predator species at time t, respectively. A. Korobeinikov [2] showed the unique positive equilibrium of system (1.1) is globally stable.

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Lian and Xu [3] proposed the following delayed Leslie-Gower model with non-monotonic functional response:

$$\begin{aligned}\dot{x} &= rx(t)\Big(1-\frac{x(t)}{k}\Big)-\frac{mx(t)}{a+x^2(t)}y(t),\\ \dot{y} &= y(t)\Big[s\Big(1-h\frac{y(t-\tau)}{x(t-\tau)+k_2}\Big)\Big].\end{aligned} \tag{1.2}$$

By choosing the delay τ as a bifurcation parameter, the authors investigated the local asymptotic stability of the positive equilibrium and existence of local Hopf bifurcations. The supercritical stable Hopf bifurcations were also found by normal form theory.

With the aim of finding how the hunting delay affects the dynamics of the Leslie-Gower predator-prey model with nonmonotonic functional response, Jiao and Song [4] proposed the following model:

$$\begin{aligned}\dot{x} &= rx(t)\Big(1-\frac{x(t)}{k}\Big)-\frac{mx(t)}{a+x^2(t)}y(t-\tau),\\ \dot{y} &= s\Big(1-\frac{y(t)}{nx(t)}\Big)y(t).\end{aligned} \tag{1.3}$$

Such topics as the existence and local stability, global stability property of the positive equilibrium of the system were investigated. The authors also investigated the Hopf bifurcations of the system. In [5], the authors further investigated the dynamic behaviors of system (1.2) near the Bogdanov-Takens bifurcation point. By analyzing the characteristic equation associated with the non-hyperbolic equilibrium, the critical value of the delay inducing the Bogdanov-Takens bifurcation was obtained. They showed that the change of delay can result in heteroclinic orbit, homoclinic orbit and unstable limit cycle.

Yin et al [6] considered the spatial heterogeneity of the predators and preys distributions, and proposed a diffusive Leslie-Gower predator-prey system with non-monotonic functional response:

$$\begin{aligned}\frac{\partial u}{\partial t} &= d_1\Delta u+u\Big(1-u-\frac{v}{au^2+bu+1}\Big), \quad x\in\Omega,\ t>0,\\ \frac{\partial v}{\partial t} &= d_2\Delta v+\eta v\Big(1-\frac{rv}{u}\Big), \quad x\in\Omega,\ t>0.\end{aligned} \tag{1.4}$$

The authors investigated the persistence of the model, the local and global stability of positive constant equilibrium and the Turing instability of the equilibrium.

On the other hand, more and more scholars have paid attention to the dynamic behaviors of the predator-prey system incorporating a prey refuge (see [7-10]). Chen et al [7] extended model (1.1) by incorporating a refuge protecting mH of the prey,