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# POSITIVE PERIODIC SOLUTIONS OF THE FIRST-ORDER SINGULAR DISCRETE SYSTEMS*†

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**Abstract**

In this paper, we study two kinds of first-order singular discrete systems. By the fixed point index theory, we investigate the existence and multiplicity of positive periodic solutions of the systems.

**Keywords** positive periodic solutions; singular discrete systems; delay; fixed point index

**2000 Mathematics Subject Classification** 34B15; 34C25

## 1 Introduction

Let $T > 3$ be an integer. In this paper, we are concerned with the existence and multiplicity of positive $T$-periodic solutions of the following singular discrete systems

$$\Delta u_i(t) = -a_i(t)g_i(\mathbf{u}(t))u_i(t) + \lambda b_i(t)f_i(\mathbf{u}(t-\tau(t))), \quad t \in \mathbb{Z},\ i = 1, 2, \cdots, n \quad (1.1)$$

and

$$\Delta u_i(t) = a_i(t)g_i(\mathbf{u}(t))u_i(t) - \lambda b_i(t)f_i(\mathbf{u}(t-\tau(t))), \quad t \in \mathbb{Z},\ i = 1, 2, \cdots, n, \quad (1.2)$$

where $\mathbf{u} = (u_1, \cdots, u_n) \in \mathbb{R}^n$, $a_i, b_i : \mathbb{Z} \to [0, \infty)$ are $T$-periodic functions with

$$\sum_{t=0}^{T-1} a_i(t) > 0, \quad \sum_{t=0}^{T-1} b_i(t) > 0;$$

$g_i \in C(\mathbb{R}^n_+, [0, \infty))$ and $f_i : \mathbb{R}^n_+ \backslash \{\mathbf{0}\} \to [0, \infty)$ are continuous, $i = 1, 2, \cdots, n$; $\tau : \mathbb{Z} \to \mathbb{Z}$ is a $T$-periodic function and $\lambda$ is a positive parameter.

In the past few years, there has been considerable interest in the existence of periodic solutions of equations

$$u'(t) = a(t)g(u(t))u(t) - \lambda b(t)f(u(t-\tau(t))) \quad (1.3)$$

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and

$$u'(t) = -a(t)g(u(t))u(t) + \lambda b(t)f(u(t-\tau(t))), \tag{1.4}$$

where $a, b \in C(\mathbb{R}, [0, \infty))$ are $T$-periodic functions with

$$\int_0^T a(t)\mathrm{d}t > 0, \quad \int_0^T b(t)\mathrm{d}t > 0,$$

and $\tau$ is a continuous $T$-periodic function. Equations (1.3) and (1.4) have been proposed as models for a variety of physiological processes and conditions including production of blood cells, respiration, and cardiac arrhythmias. See for example, [1-8,12] and the references therein. On the other hand, many authors paid their attention to the existence of positive periodic solutions of singular systems of the first-order and second-order differential equations, see Chu [9], Jiang [10], Wang [11,12] and the references therein. It has been shown that many results of nonsingular systems still valid for singular cases.

Let

$$\mathbb{R}_+ = [0, \infty), \quad \mathbb{R}_+^n = \prod_{i=1}^n \mathbb{R}_+,$$

and for any $\mathbf{u} = (u_1, \cdots, u_n) \in \mathbb{R}_+^n$,

$$\|\mathbf{u}\| = \sum_{i=1}^n |u_i|.$$

Recently, Wang [12] studied the existence and multiplicity of positive periodic solutions of the following singular non-autonomous $n$-dimensional system

$$x_i'(t) = -a_i(t)x_i(t) + \lambda b_i(t)f_i(x_1(t), \cdots, x_n(t)), \quad i = 1, \cdots, n \tag{1.5}$$

under assumptions

(H1) $a_i, b_i \in C(\mathbb{R}, [0, \infty))$ are $\omega$-periodic functions such that $\int_0^\omega a_i(t)\mathrm{d}t > 0$, $\int_0^\omega b_i(t)\mathrm{d}t > 0$, $i = 1, \cdots, n$;

(H2) $f_i : \mathbb{R}_+^n \backslash \{\mathbf{0}\} \to (0, \infty)$ are continuous, $i = 1, \cdots, n$.

By using Krasnoselskii fixed point theorem in a cone, the author established the existence and multiplicity of positive periodic solutions of (1.5) with superlinearity or sublinearity assumptions at infinity for an appropriately chosen parameter.

However, to the best of our knowledge, the existence results of positive periodic solutions for first-order discrete systems (1.1) and (1.2) with singular nonlinearities are relatively little. Motivated by the above considerations, in this paper, we study the existence and multiplicity of positive $T$-periodic solutions of singular discrete systems (1.1) and (1.2). Obviously, (1.1) is a discrete analogue of system (1.5) when $g_i \equiv 1$, $i = 1, 2, \cdots, n$ and $\tau \equiv 0$, and we are interested in establishing the similar results as [12, Theorem 1.1] for systems (1.1) and (1.2).