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ON THE CONDITIONAL EDGE CONNECTIVITY OF ENHANCED HYPERCUBE NETWORKS*†

Yanjuan Zhang‡, Hongmei Liu, Dan Jin
(*College of Science China Three Gorges University, Yichang 443002, Hubei, PR China*)

Abstract

Let $G = (V, E)$ be a connected graph and m be a positive integer, the conditional edge connectivity λ_δ^m is the minimum cardinality of a set of edges, if it exists, whose deletion disconnects G and leaves each remaining component with minimum degree δ no less than m. This study shows that $\lambda_\delta^1(Q_{n,k}) = 2n$, $\lambda_\delta^2(Q_{n,k}) = 4n - 4$ $(2 \leq k \leq n-1,\ n \geq 3)$ for n-dimensional enhanced hypercube $Q_{n,k}$. Meanwhile, another easy proof about $\lambda_\delta^2(Q_n) = 4n - 8$, for $n \geq 3$ is proposed. The results of enhanced hypercube include the cases of folded hypercube.

Keywords interconnected networks; connectivity; conditional edge connectivity; fault tolerance; enhanced hypercube

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1 Introduction

A multiprocessor system may contain numerous number of nodes, some of which may be faulty when the system is implemented. Reliability and fault tolerance are two of the most critical concerns of multiprocessor systems. Based on the definition proposed by Esfahanian [1], a multiprocessor system is fault tolerance if it can remain functional when failures occur. Two basic functionality criteria have received many attention. The first one is whether the network logically contains a certain topological structure. This problem occurs when embedding one architecture into another. This approach involves system wide redundancy and reconfiguration. The second functionality criterion considers a multiprocessor system function if a fault-free path exists between any two fault-free nodes. Hence, connectivity and edge connectivity are two important measurements of this criterion [2]. A vertex cut of a connected graph G is a set of vertices whose removal disconnects G. The connectivity $\kappa(G)$ of a connected graph G is the cardinality of a minimum vertex cut. An edge

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†Manuscript received February 24, 2016; Revised May 30, 2018
‡Corresponding author. E-mail: pyyj134@163.com

cut of a connected graph G is a set of edges whose removal disconnects G. The edge connectivity $\lambda(G)$ of a connected graph G is the cardinality of a minimum edge cut [3]. However this two parameters may result in an isolated vertex. This is practically impossible in some network applications. To address this deficiency, many specific terms forbidden fault set and forbidden fault edge set are introduced such as conditional connectivity [4], extra connectivity and extra edge connectivity [5].

The n-dimensional hypercube Q_n is one of the most versatile and efficient interconnected networks because of its regular structure, small diameter, and good connection with a relative small node degree [2], all of which are very important for designing parallel systems. As the importance of hypercubes, many variants of Q_n have been proposed, among which, for instance, are crossed hypercube, argument hypercube, folded hypercube and enhanced hypercube. As an enhancement on the hypercube Q_n, the enhanced hypercube $Q_{n,k}$ proposed by Tzeng and Wei [6], not only retains some of the favorable properties of Q_n, but also improve the efficiency of the hypercube structure, since it possesses many properties superior to hypercube [7-9]. For example, the diameter of the enhanced hypercube is almost half of the hypercube. The hypercube is n-regular and n-connected, whereas the enhanced hypercube is $(n+1)$-regular and $(n+1)$-connected. Its special case of $k=1$ is the well-known Folded hypercube (denoted by FQ_n), which has been used as underlying topologies of several parallel systems, such as ATM switches [10,11] PM2I networks [12], and 3D-FoIHNOC networks [13] for high-speed cell-switching and reducing the diameter and traffic congestion of the hypercube with little hardware overhead.

The conditional edge connectivity λ_δ^m which is the generalization of edge connectivity, is defined as the cardinality of the minimum edge cut, if it exists, whose deletion disconnects G and leaves each component with minimum degree δ no less than m. Xu [2] provided $\kappa(Q_n) = \lambda(Qn) = n$, $\kappa(FQ_n) = \lambda(FQ_n) = n+1$ and $\kappa(Q_{n,k}) = \lambda(Q_{n,k}) = n+1$. Obviously, $\lambda_e^0(G) = \lambda(G)$. Paper [14] made a good job on the conditional edge connectivity of Q_n and proved that $\lambda_\delta^1(Q_n) = 2n-2$ for $n \geq 2$, $\lambda_\delta^2(Q_n) = 4n-8$ for $n \geq 3$ and $\lambda_\delta^2(FQ_n) = 4n-4$ for $n \geq 4$. However there is nothing about the conditional edge connectivity of enhanced hypercube. In this paper, we discuss the properties of enhanced hypercubes $Q_{n,k}$ and show that $\lambda_\delta^1(Q_{n,k}) = 2n$, $\lambda_\delta^2(Q_{n,k}) = 4n-4$ for $n \geq 3$ which includes the results of folded hypercube. Meanwhile, this paper proposes an easy proof of $\lambda_\delta^2(Q_n) = 4n-8$ for $n \geq 3$.

2 Preliminaries

The graph theoretical definitions and notations follow [2]. A network is usually modeled by a connected graph $G = (V, E)$, where V and E represent the vertex and