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RAMSEY NUMBER OF HYPERGRAPH PATHS*

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Abstract

Let $H=(V,E)$ be a k-uniform hypergraph. For $1\le s\le k-1$, an s-path $P_n^{(k,s)}$ of length n in H is a sequence of distinct vertices $v_1,v_2,\cdots,v_{s+n(k-s)}$ such that $\{v_{1+i(k-s)},\cdots,v_{s+(i+1)(k-s)}\}$ is an edge of H for each $0\le i\le n-1$. In this paper, we prove that $R(P_n^{(3s,s)},P_3^{(3s,s)})=(2n+1)s+1$ for $n\ge 3$.

Keywords hypergraph Ramsey number; path
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1 Introduction

A k-uniform hypergraph H with vertex V is a collection of k-element subset of V. We write $K_n^{(k)}$ for the complete k-uniform hypergraph on an n-element vertex set. For given two k-uniform hypergraphs H_1 and H_2, the Ramsey number $R(H_1,H_2)$ is defined to be the minimum value of N such that each red-blue coloring of edges of $K_N^{(k)}$ contains either a monochromatic red copy of H_1, or a monochromatic blue copy of H_2. Let H be a k-uniform hypergraph. For each $1\le s\le k-1$, an s-path $P_n^{(k,s)}$ of length n in H is a sequence of distinct vertices $v_1,v_2,\cdots,v_{s+n(k-s)}$ such that $\{v_{1+i(k-s)},\cdots,v_{s+(i+1)(k-s)}\}$ is an edge e_{i+1} of H for each $0\le i\le n-1$. We also say that the edges $e_1,e_2,\cdots,e_n$ form a path $P_n^{(k,s)}$. Similarly, an s-cycle $C_n^{(k,s)}$ of length n in H is a sequence of vertices $v_1,v_2,\cdots,v_{s+n(k-s)}$ such that $\{v_{1+i(k-s)},\cdots,v_{s+(i+1)(k-s)}\}$ is an edge of H for each $0\le i\le n-1$, $v_1,v_2,\cdots,v_{n(k-s)}$ are distinct, and $v_{j+n(k-s)}=v_j$ for each $1\le j\le s$.

When $k=2$ and $s=1$, a classical result by Gerencsér and Gyárfás [4] is $R(P_n,P_m)=n+\lfloor\frac{m+1}{2}\rfloor$ for $n\ge m\ge 1$; it is also known from [2,3] that $R(P_n,C_m)=R(P_n,P_m)=n+\frac{m}{2}$ for $n\ge m$ with m being even. Recently, the hypergraph Ramsey numbers also attract lots of attention. When $s=1$, Haxell et al. [5] first determined that the asymptotic values of $R(P_n^{(3,1)},P_n^{(3,1)})$, $R(P_n^{(3,1)},C_n^{(3,1)})$ and $R(C_n^{(3,1)},C_n^{(3,1)})$

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are $\frac{5n}{2}$. Later, Gyárfás, Sárközy and Szemenrédi [6] extended this result to all $k \geq 3$. Namely, they proved that $R(P_n^{(k,1)}, P_n^{(k,1)})$, $R(P_n^{(k,1)}, C_n^{(k,1)})$, $R(C_n^{(k,1)}, C_n^{(k,1)})$ are asymptotically equal to $\frac{(2k-1)n}{2}$. There are some exact results on short paths and cycles. Gyárfás and Raeisi [7] proved that

$$R(P_3^{(k,1)}, P_3^{(k,1)}) = R(P_3^{(k,1)}, C_3^{(k,1)}) = R(C_3^{(k,1)}, C_3^{(k,1)}) + 1 = 3k - 1$$

and

$$R(P_4^{(k,1)}, P_4^{(k,1)}) = R(P_4^{(k,1)}, C_4^{(k,1)}) = R(C_4^{(k,1)}, C_4^{(k,1)}) + 1 = 4k - 2.$$

Recently, Omidi and Shahsiah [8] raised the conjecture that

$$R(P_n^{(k,1)}, P_m^{(k,1)}) = R(P_n^{(k,1)}, C_m^{(k,1)}) = R(C_n^{(k,1)}, C_m^{(k,1)}) + 1 = (k-1)n + \frac{m+1}{2}$$

is equivalent to

$$R(C_n^{(k,1)}, C_m^{(k,1)}) + 1 = (k-1)n + \frac{m-1}{2} \quad \text{for } k = 3.$$

Later, the authors showed that for fixed $m \geq 3$ and $k \geq 4$ the former is equivalent to (only) the last equality of the latter for any $2m \geq n \geq m \geq 3$. More precisely, they proved that for fixed $m \geq 3$ and $k \geq 4$, the latter is true for each $n \geq m$ if and only if it is true for the former for $2m \geq n \geq m \geq 3$. In 2016, Peng [1] proved that for $s \geq 1$ and $n \geq 3$,

$$R(P_n^{(2s,s)}, P_3^{(2s,s)}) = (n+1)s + 1;$$

and for $s \geq 1$ and $n \geq 4$,

$$R(P_n^{(2s,s)}, P_4^{(2s,s)}) = (n+1)s + 1.$$

A general lower bound is as follows.

Lemma 1[5] *For each $n \geq m \geq 1$ and $1 \leq s \leq k/2$, we have*

$$R(P_n^{(k,s)}, P_m^{(k,s)}) > s + n(k-s) + \left\lfloor \frac{m+1}{2} \right\rfloor - 2.$$

In this paper, we mainly consider the case of $k = 3s$. In order to avoid the excessive use of superscripts, we use the simpler notations

$$R(P_n^{(3s,s)}, P_m^{(3s,s)}) = R(P_n, P_m) \quad \text{and} \quad P_n^{(3s,s)} = P_n.$$

In this note, we have the following result.

Theorem 1 *For each $s \geq 1$ and $n \geq 3$, $R(P_n, P_3) = (2n+1)s + 1$.*