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HYBRID ITERATION METHOD FOR GENERALIZED EQUILIBRIUM PROBLEMS AND FIXED POINT PROBLEMS OF A FINITE FAMILY OF ASYMPTOTICALLY NONEXPANSIVE MAPPINGS[*†]

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Abstract

In this paper, weak and strong convergence theorems are established by hybrid iteration method for generalized equilibrium problem and fixed point problems of a finite family of asymptotically nonexpansive mappings in Hilbert spaces. The results presented in this paper partly extend and improve the corresponding results of the previous papers.

Keywords generalized equilibrium problem; a finite family of asymptotically nonexpansive mapping; hybrid iteration method; inverse-strongly monotone mapping; Hilbert space

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1 Introduction

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$. Let C be a nonempty closed convex subset of H, $G : C\times C \to R$ be a bifunction and $A : C \to H$ be a nonlinear mapping. The generalized equilibrium problem (for short, GEP) is to find an $x \in C$ such that

$$G(x, y) + \langle Ax, y - x\rangle \geq 0, \quad \text{for any } y \in C. \tag{1.1}$$

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The set of solutions of (1.1) is denoted by $GEP(G)$, that is,

$$GEP(G) = \{x \in C : G(x, y) + \langle Ax, y - x\rangle \geq 0, \text{ for any } y \in C\}.$$

In the case of $A \equiv 0$, GEP is denoted by $EP(G)$. In the case of $G \equiv 0$, GEP is also denoted by $VI(C, A)$. Problem (1.1) covers monotone inclusion problems, saddle point problems, minimization problems, optimization problems, variational inequality problems, and Nash equilibria in noncooperative games. Recently, many authors have studied the problem of finding a common element of the set of fixed points of a nonexpansive mapping and the set of an equilibrium problem in the framework of Hilbert spaces and Banach spaces, respectively; see, for instance, [1-8] and the references therein.

In 2007, to study the strong and weak convergence of fixed points of nonexpansive mappings, Wang [9] introduced the following hybrid iteration scheme:

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)T^{\lambda_{n+1}} x_n, \quad n \geq 0, \tag{1.2}$$

where $T^{\lambda}x = Tx - \lambda\mu F(Tx)$ for all $x \in H$ and $x_0 \in H$ is chosen arbitrarily, $F : H \to H$ is an η-strongly monotone and k-Lipschitzian, they obtained that under some suitable conditions, the sequence $\{x_n\}$ converges weakly to a fixed point of T, and under the necessary and sufficient conditions, $\{x_n\}$ converges strongly to a fixed point of T.

In 2009, Ceng and Gao [3] introduced the following iterative scheme for a k-strict pseudo-contraction mapping in Hilbert space: $x_1 = x \in H$,

$$\begin{cases} G(u_n, y) + \dfrac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \geq 0, \quad \text{for any } y \in C, \\ x_{n+1} = \alpha_n u_n + (1 - \alpha_n)Tu_n, \end{cases} \tag{1.3}$$

for every $n \in N$, where $\alpha_n \subset [a, b]$ for some $a, b \in (k, 1)$ and $\{r_n\} \subset (0, \infty)$ satisfies $\liminf_{n\to\infty} r_n > 0$. Further, they proved $\{x_n\}$ and $\{u_n\}$ converge weakly to $z \in F(T) \cap EP(G)$, where $z = P_{F(T)\cap EP(G)}x$.

In 2012, Wang and Huang [10] considered hybrid iteration method for a finite asymptotically nonexpansive mappings in Banach space:

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)[T_{i(n)}^{k_(n)} x_n - \lambda_{n+1}\mu f(T_{i(n)}^{k_(n)} x_n)], \quad \text{for any } n \geq 1, \tag{1.4}$$

where $x_1 \in E$ is chosen arbitrarily, $\{T_1, T_2, \cdots, T_N\} : K \to K$ are N asymptotically nonexpansive mappings, $f : K \to K$ is a L-Lipschitzian mapping. And weak and strong convergence theorems are obtained under some suitable conditions.

Motivated by those works of [3] and [10], in this paper we introduce the following hybrid iteration method: $x_1 \in C$