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GLOBAL HOPF BIFURCATION IN A DELAYED PHYTOPLANKTON-ZOOPLANKTON MODEL WITH COMPETITION*

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Abstract

In this paper, the dynamics of a delayed phytoplankton-zooplankton model is considered. Taking the delay due to the gestation of zooplankton as parameter, we describe the local Hopf bifurcation by center manifold theorem and normal form, then we discuss the global existence of periodic solution. At last, some simulations are given to support our result.

Keywords global Hopf bifurcation; normal form; periodic solution; competition; phytoplankton-zooplankton

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1 Introduction

Recently, there have been many works about the phytoplankton-zooplankton model [1-5,7]. The model is important for aquatic environment. The phytoplankton could produce much oxygen and absorb much carbon dioxide, they benefit our environment very much. As we know, some phytoplankton could be harmful for zooplankton, they could create toxin substance which could kill the aquatic animals. From [1,4,5], we know that the delay caused by the maturity of toxic-phytoplankton plays an important role on the dynamic of phytoplankton-zooplankton system, which seems that delay could cause rich dynamics. In [4], the author considered two harmful phytoplankton-zooplankton model with two delays

$$\begin{cases} \dfrac{\mathrm{d}P_1}{\mathrm{d}t} = r_1P_1\Big(1-\dfrac{P_1}{K}\Big) - \alpha_1P_1P_2 - \rho_1P_1Z, \\ \dfrac{\mathrm{d}P_2}{\mathrm{d}t} = r_2P_2\Big(1-\dfrac{P_2}{K}\Big) - \alpha_2P_1P_2 - \rho_2P_2Z, \\ \dfrac{\mathrm{d}Z}{\mathrm{d}t} = (r_1P_1+r_2P_2)Z - dZ - \theta_1P_1(t-\tau_1)Z - \theta_2P_2(t-\tau_2)Z, \end{cases} \tag{1.1}$$

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where local Hopf bifurcation was discussed with two different delays. In the delayed two zooplankton-phytoplankton model with competition [1]

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}t} = rP\Big(1-\dfrac{P}{K}\Big) - \dfrac{\mu_1 P Z_1}{\alpha_1+P} - \dfrac{\mu_2 P Z_1}{\alpha_2+P}, \\ \dfrac{\mathrm{d}Z_1}{\mathrm{d}t} = \dfrac{\beta_1 P Z_1}{\alpha_1+P} - \dfrac{\rho_1 P(t-\tau)Z_1}{\alpha_1+P(t-\tau)} - d_1 Z_1 - g_1 Z_1^2, \\ \dfrac{\mathrm{d}Z_2}{\mathrm{d}t} = \dfrac{\beta_2 P Z_1}{\alpha_2+P} - \dfrac{\rho_2 P(t-\tau)Z_2}{\alpha_2+P(t-\tau)} - d_2 Z_2 - g_1 Z_2^2. \end{cases} \tag{1.2}$$

The authors discussed the local Hopf bifurcation under taking delay τ as the parameter. As we know, when the delay τ is located in a sufficiently small neighborhood of the critical value, local hopf bifurcation occurs. But it is difficult to show the global existence of periodic solution. The works about the global Hopf bifurcation of phytoplankton-zooplankton system have been obtained in recent years, such as [2,3,7]. In [2], the authors assumed the delay of gestation equals the delay required for maturity of toxic phytoplankton. The global Hopf bifurcation of the following system was discussed

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}t} = rP(t)\Big(1-\dfrac{P(t)}{K}\Big) - \dfrac{\beta P(t)Z(t)}{1+\gamma_1 P(t)}, \\ \dfrac{\mathrm{d}Z}{\mathrm{d}t} = \dfrac{\mathrm{e}^{-\delta\tau_1}\beta_1 P(t-\tau_1)Z(t-\tau_1)}{1+\gamma_1 P(t-\tau_1)} - \delta Z(t) - \dfrac{\mathrm{e}^{-\delta\tau_2}\rho P(t-\tau_2)Z(t-\tau_2)}{1+\gamma_2 P(t-\tau_2)}. \end{cases} \tag{1.3}$$

In [7], the authors only considered the delay caused by the gestation of zooplankton, and the global Hopf bifurcation was discussed,

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}t} = rP\Big(1-\dfrac{P}{K}\Big) - \alpha PZ, \\ \dfrac{\mathrm{d}Z}{\mathrm{d}t} = \beta P(t-\tau)Z(t-\tau) - \mu Z - \dfrac{\theta PZ}{\gamma+P}. \end{cases} \tag{1.4}$$

Besides, there have been other works about the global Hopf bifurcation [8,9]. In our opinion, competition is a common phenomena in nature, so we take competition into the zooplankton, model (1.4) becomes

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}t} = rP\Big(1-\dfrac{P}{K}\Big) - \dfrac{\mu PZ}{\alpha+P}, \\ \dfrac{\mathrm{d}Z}{\mathrm{d}t} = \dfrac{\beta P(t-\tau)Z(t-\tau)}{\alpha+P(t-\tau)} - \dfrac{\rho P(t)Z(t)}{\alpha+P(t)} - dZ - gZ^2, \end{cases} \tag{1.5}$$

where P and Z denote the densities of the phytoplankton and zooplankton respectively, r denotes the intrinsic growth rate, and K is the environmental carrying