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GLOBAL ATTRACTIVITY IN AN ALMOST PERIODIC PREDATOR-PREY-MUTUALIST SYSTEM*†

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Abstract

In this paper, the almost periodic predator-prey-mutualist model with Holling type II functional response is discussed. A set of sufficient conditions which guarantee the uniform persistence and the global attractivity of the system are obtained. For the almost periodic case, by constructing a suitable Lyapunov function, sufficient conditions which guarantee the existence of a unique globally attractive positive almost periodic solution of the system are obtained. An example together with its numerical simulations shows the feasibility of the main results.

Keywords almost periodic solution; predator-prey-mutualist system; functional response; Lyapunov function; global attractivity

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1 Introduction

As was pointed out by Berryman [1], the dynamic relationship between predator and prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Already, the predator-prey model has been studied by several scholars [2-10]. For example, Das etc. [8] investigated a three species ecosystem consisting of a prey, predator and a top predator. They derived the criteria for local and global stability of all the eight equilibrium points using Routh-Hurwitz and Lyapunov function. Wu and Li [9] studied the permanence and global attractivity of the discrete predator-

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prey system with Hassell-Varley-Holling III type functional response. Chen and Chen [10] proposed a ratio-dependent predator-prey model incorporating a prey refuge. They studied the global stability, limit cycle and Hopf bifurcation of the system.

Though mutualism is one of the most important relationships in the real world, for instance, ants prevent herbivores from feeding on plants (see [11]) and ants prevent predators from feeding on aphids (see [12,13]). As was pointed out by Murray [14]: "this area has not been as widely studied as the others even though its importance is comparable to that of predator-prey and competition interactions." To this end, Rai and Krawcewicz [15] proposed the following three species predator-prey-mutualist system:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \alpha x\Big(1 - \dfrac{x}{K}\Big) - \dfrac{\beta x z}{1 + my}, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = \gamma y\Big(1 - \dfrac{y}{lx + L_0}\Big), \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = z\Big(-s + \dfrac{c\beta x}{1 + my}\Big), \end{cases} \tag{1.1}$$

where $x(t)$, $y(t)$ and $z(t)$ denote the densities of prey, mutualist and predator population at any time t, respectively. They applied the equivariant degree method to study Hopf bifurcations phenomenon of the system.

Recently, Yang, Xie and Wu [16] argued that due to seasonal effects of weather, temperature, food supply, mating habits etc, a more appropriate system should be the non-autonomous case, and they proposed and studied the following system:

$$\begin{cases} \dot{x} = x\Big(a_1(t) - b_1(t)x - \dfrac{c_1(t)z}{d_1(t) + d_2(t)y}\Big), \\ \dot{y} = y\Big(a_2(t) - \dfrac{y}{d_3(t) + d_4(t)x}\Big), \\ \dot{z} = z\Big(-a_3(t) + \dfrac{k_1(t)c_1(t)x}{d_1(t) + d_2(t)y} - b_2(t)z\Big). \end{cases} \tag{1.2}$$

By using the Brouwer fixed pointed theorem and constructing a suitable Lyapunov function, the authors obtained a set of sufficient conditions for the existence of a globally asymptotically stable periodic solution of system (1.2).

It brings to our attention that in systems (1.1) and (1.2), the authors did not consider the functional response of the predator species, which motivates us to study a suitable predator-prey system incorporating some functional response of the predator species, and to propose the following three species predator-prey-mutualist system: