Ann. of Appl. Math.
32:3(2016), 296-310

ALTERING CONNECTIVITY WITH LARGE DEFORMATION MESH FOR LAGRANGIAN METHOD AND ITS APPLICATION IN MULTIPLE MATERIAL SIMULATION*†

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Abstract

A new approach for treating the mesh with Lagrangian scheme of finite volume method is presented. It has been proved that classical Lagrangian method is difficult to cope with large deformation in tracking material particles due to severe distortion of cells, and the changing connectivity of the mesh seems especially attractive for solving such issues. The mesh with large deformation based on computational geometry is optimized by using new method. This paper develops a processing system for arbitrary polygonal unstructured grid, the intelligent variable grid neighborhood technologies is utilized to improve the quality of mesh in calculation process, and arbitrary polygonal mesh is used in the Lagrangian finite volume scheme. The performance of the new method is demonstrated through series of numerical examples, and the simulation capability is efficiently presented in coping with the systems with large deformations.

Keywords Lagrangian scheme; large deformations; altering connectivity of mesh; finite volume method

2000 Mathematics Subject Classification 74S10

1 Introduction

The relationship between the distortion of the computational cells and the motion of the fluid plays an important role in numerical simulation of multidimensional compressible flow. Lagrangian or Eulerian coordinate is utilized to resolve this problem. However, there is a great difference between these two methods. For instance,

*The work was supported in part by the National Natural Science Foundation of China under Grant 11372051, Grant 11475029, in part by the Fund of the China Academy of Engineering Physics under Grant 20150202045.

†Manuscript received April 21, 2016

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the computational grid moves with the local fluid velocity in the Lagrangian description, while the grid is fixed in the Eulerian framework. The Lagrangian methods are prevalent in compressible multi-material fluid dynamics with high temperature and pressure, since this approach has well-resolved material interfaces and does not have advective diffusion term. However a large shearing distortion may lead to severely distorted cells and, inevitably, to mesh tangling, which will reduce the accuracy of the discrete scheme based on the grid, and the computation will run termination. Therefore, to resolve the large deformation is one difficulty and the focus in Lagrangian methods, and also the front field in CFD at present.

The deformation of the Lagrangian methods originates from two sources: One is un-robustness of the numerical scheme, and the other is grid evolution following the fluid. Thus highly qualified grid and robust scheme must be explored in order to make the Lagrangian methods having strong adaptation.

In this paper we propose an automatically local remeshing method based on changing connectivity of the mesh, which can be used to handle geometric intersection. It consists of two parts. First an arbitrary unstructured polygonal mesh is constructed through a collection of control nodes that are topologically organized into cells. By the way, the mesh is unstructured in the sense that individual cells may be constructed from arbitrary, non-uniform number of nodes. Second, changing connectivity of the mesh (topology transformation) is allowed during numerical simulation. Topological operations such as splitting and elimination of cells and edges, merging of cells is allowed in this process. This approach has successfully been implemented in a number of 2D codes of numerical analysis.

2 Governing Equations

Consider a two-dimensional multi-material compressible fluid system with elastic and plastic terms written in Lagrangian formalism given by:

$$\begin{aligned}
&\frac{\dot{V}}{V} = \dot{\epsilon}_x + \dot{\epsilon}_y + \dot{\epsilon}_\theta, \\
&\rho\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\partial \Sigma_x}{\partial x} + \frac{\partial T_{xr}}{\partial r} + \nu\frac{T_{xr}}{r}, \\
&\rho\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\partial T_{xr}}{\partial x} + \frac{\partial \Sigma_r}{\partial r} + \nu\frac{\Sigma_r - \Sigma_\theta}{r}, \\
&\rho\frac{\mathrm{d}e}{\mathrm{d}t} = \Sigma_x\dot{\epsilon}_x + \Sigma_r\dot{\epsilon}_y + \Sigma_\theta\dot{\epsilon}_z + T_{xr}\dot{\gamma}_{xr},
\end{aligned} \tag{1}$$

where x and r denote axes. In 2D planar problem, x is the level direction, r is the perpendicular direction. In cylindrical symmetry model, x is the axial symmetry direction, r is the perpendicular direction. u, v, ρ, e and p are x-velocity component,