Mesh-Free Interpolant Observables for Continuous Data Assimilation

Animikh Biswas[1], Kenneth R. Brown[2] and Vincent R. Martinez[3,4,*]

[1] *Department of Mathematics & Statistics, University of Maryland–Baltimore County, 1000 Hilltop Circle, Baltimore, MD 21250, USA*
[2] *Department of Mathematics, University of California–Davis, One Shields Avenue, Davis, CA 95616, USA*
[3] *Department of Mathematics & Statistics, CUNY Hunter College, 695 Park Ave, New York, NY 10065, USA*
[4] *Department of Mathematics, CUNY Graduate Center, 365 5th Ave, New York, NY 10016, USA*

Abstract. This paper is dedicated to the expansion of the framework of general interpolant observables introduced by Azouani, Olson, and Titi for continuous data assimilation of nonlinear partial differential equations. The main feature of this expanded framework is its mesh-free aspect, which allows the observational data itself to dictate the subdivision of the domain via partition of unity in the spirit of the so-called Partition of Unity Method by Babuska and Melenk. As an application of this framework, we consider a nudging-based scheme for data assimilation applied to the context of the two-dimensional Navier-Stokes equations as a paradigmatic example and establish convergence to the reference solution in all higher-order Sobolev topologies in a periodic, mean-free setting. The convergence analysis also makes use of absorbing ball bounds in higher-order Sobolev norms, for which explicit bounds appear to be available in the literature only up to H^2; such bounds are additionally proved for all integer levels of Sobolev regularity above H^2.

*Corresponding author.
Emails: abiswas@umbc.edu (A. Biswas), kbr@ucdavis.edu (K. Brown), vrmartinez@hunter.cuny.edu (V. Martinez)

1   Introduction

In recent years, several efforts have been made to develop a first-principles understanding of Data Assimilation (DA), where the underlying model dynamics are given by partial differential equations (PDEs) [2,6,8,11,13,20,39,40,54,56,57], as well to provide rigorous analytical and computational justification for its application and support for common practices therein, especially in the context of numerical weather prediction [1,3–5,24–30,32,36,38,41,42,50–52,55]. A common representative model in these studies is the forced, two-dimensional (2D) Navier-Stokes equations (NSE) of an incompressible fluid, which contains the difficulty of high-dimensionality by virtue of being an infinite-dimensional, chaotic dynamical system, but whose long-time dynamics is nevertheless finite-dimensional, manifested, for instance, in the existence of a finite-dimensional global attractor [16,31,59]. Given a domain $\Omega \subset \mathbb{R}^2$, the 2D NSE is given by

$$\partial_t u + (u \cdot \nabla)u = -\nabla p + \nu \Delta u + f, \qquad \nabla \cdot u = 0, \tag{1.1}$$

supplemented with appropriate boundary conditions, where u represents the velocity vector field, ν denotes the kinematic viscosity, f is a time-independent, external driving force, p represents the scalar pressure field. The underlying ideas in the works above, though originally motivated in large part by the classical problem of DA, that is, of reconstructing the underlying reference signal, has since been extended to the problem of parameter estimation; we refer the readers to the recent works [17,18,53] for this novel application.

   Central to the investigations of this paper is a certain algorithm for DA which synchronizes the approximating signal produced by the algorithm with the true signal corresponding to the observations. The algorithm of interest in this paper is a nudging-based scheme in which observational data of the signal is appropriately extended to the phase space of the system representing the truth, (1.1). The interpolated data is then inserted into the system as an exogeneous term and is subsequently balanced through a feedback control term that serves to drive the approximating signal towards the observations. In particular, we consider the approximating signal to be given as a solution to the system

$$\partial_t v + (v \cdot \nabla)v = -\nabla q + \nu \Delta v + f - \mu(I_h v - I_h u), \quad \nabla \cdot v = 0, \tag{1.2}$$