Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **15**, No. 1, pp. 49-68

DOI: 10.4208/aamm.OA-2021-0381
February 2023

Finite-Time Stability and Instability of Nonlinear Impulsive Systems

Guihua Zhao[1] and Hui Liang[2,*]

[1] *College of Science, University of Shanghai for Science and Technology, Shanghai, 200093, China*
[2] *School of Science, Harbin Institute of Technology, Shenzhen, Guangdong 518055, China*

Received 23 December 2021; Accepted (in revised version) 4 March 2022

Abstract. In this paper, the finite-time stability and instability are studied for nonlinear impulsive systems. There are mainly four concerns. 1) For the system with stabilizing impulses, a Lyapunov theorem on global finite-time stability is presented. 2) When the system without impulsive effects is globally finite-time stable (GFTS) and the settling time is continuous at the origin, it is proved that it is still GFTS over any class of impulse sequences, if the mixed impulsive jumps satisfy some mild conditions. 3) For systems with destabilizing impulses, it is shown that to be finite-time stable, the destabilizing impulses should not occur too frequently, otherwise, the origin of the impulsive system is finite-time instable, which are formulated by average dwell time (ADT) conditions respectively. 4) A theorem on finite-time instability is provided for system with stabilizing impulses. For each GFTS theorem of impulsive systems considered in this paper, the upper boundedness of settling time is given, which depends on the initial value and impulsive effects. Some numerical examples are given to illustrate the theoretical analysis.

AMS subject classifications: 34A37, 93D40

Key words: Impulsive systems, finite-time stability, finite-time instability, nonlinear systems.

1 Introduction

Impulsive systems combine continuous evolution (typically modelled by ordinary differential equations) with instantaneous state jumps or resets (also referred to as impulses) (see [6]). Due to the rich applications, impulsive systems have attracted lots of researchers' attention (e.g., [1,3,5,9,11–14,16,17,23,24,26–28]).

*Corresponding author.
Emails: zgh-hit1108@163.com (G. Zhao), lianghui@hit.edu.cn; wise2peak@126.com (H. Liang)

Recently, finite-time stability of impulsive systems has also attracted lots of attention (e.g., [3, 11, 17–22]), because comparing with infinite-time stable system, the finite-time stable system has faster convergence, better robustness and better disturbance rejection (see [2, 25]). Meanwhile, finite-time stability has rich applications in practical systems, such as spacecraft system [4], continuously stirred tank reactor system [7] and mechanical system [10]. In [3], by using the existing results about continuous systems, the finite-time stability of nonlinear impulsive systems is obtained; in [17], the finite-time stability for systems with stabilizing impulses and the finite-time stabilization of impulsive dynamical systems are studied; in [11, 19–22], the finite-time stability of system with stabilizing impulses and finite-time stability of system with destabilizing impulses are studied, and upper boundedness of settling time is estimated; in [18], for the coupled impulsive neural networks with time-varying delays and saturating actuators, the finite-time stabilization is achieved by using the finite-time stability theorem in [11, Theorem 1], which is a special case of Theorem 3.1 of this paper.

On the one hand, in [3, 11, 17], the original systems without impulsive effects is required to be finite-time stable. On the other hand, the upper right-hand Dini derivative of the Lyapunov function in the finite-time stability theorems depends on the initial value and the first impulsive time (e.g., [21]) or a finite-time stable function pair (e.g., [19, 20, 22]). Hence, this agrees us to study the finite-time stability for more general impulsive systems. In our finite-time stability theorems, the system without impulsive effects may not be finite-time stable, the upper right-hand Dini derivative of the Lyapunov function may not be related to the finite-time stable function pair and may be independent of the initial value and the first impulsive time. Because the impulsive events occur in a finite or infinite sequence of time (see [24]), we study the finite-time stability for the nonlinear impulsive systems, whose impulse sequence may be finite or infinite, instead of only finite impulse sequence (e.g., [19, 21, 22]) or only infinite impulse sequence (e.g., [20]). Besides, the results about finite-time stability of impulsive systems in [11, 17] are involved in our results.

For the system with stabilizing impulses, we provide a Lyapunov theorem on the global finite-time stability. It is shown that the more frequently the impulses occur, the faster the system state reaches the origin, which is formulated by an ADT condition. For systems with mixed impulses (some impulses are stabilizing and some are destabilizing), we show that GFTS system with settling time being continuous at the origin is still GFTS under impulsive effects, if the jumps satisfy some mild conditions. Next, we give a rigorous proof on the global finite-time stability of system with destabilizing impulses. It is shown that the impulses should not occur too frequently, otherwise the origin of the impulsive system is finite-time instable, which are formulated by ADT conditions respectively. In addition, the finite-time instability of system with stabilizing impulses is also studied.

The main contributions of this paper include: 1) Our results can be applied to more general impulsive systems, since our results not only cover all results about the finite-time stability of impulsive systems in [11, 17], but also cover some systems whose finite-