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Approximation of Functionals by Neural Network Without Curse of Dimensionality

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Summary for general readers:

Learning functionals or operators by neural networks is nowadays widely used in computational and applied mathematics. Compared with learning functions by neural networks, an essential difference is that the input spaces of functionals or operators are infinite dimensional space. Some recent works learnt functionals or operators by reducing the input space into a finite dimensional space. However, the curse of dimensionality always exists in this type of methods. That is, in order to maintain the accuracy of an approximation, the number of sample points grows exponentially with the increase of dimension.

In this paper, we establish a new method for the approximation of functionals by neural networks without curse of dimensionality. Functionals, such as linear functionals and energy functionals, have a wide range of important applications in science and engineering fields. We define Fourier series of functionals and the associated Barron spectral space of functionals, based on which our new neural network approximation method is established. The parameters and the network structure in our method only depend on the functional. The approximation error of the neural network is $O(1/\sqrt{m})$ where m is the size of the network, which does not depend on the dimensionality.

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