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A Theoretically Complete Surface Segmentation Method for CNC Subtractive Fabrication

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Abstract. We present a well improved surface segmentation algorithm for 3-axis/3+2-axis CNC subtractive fabrication. For a free-form surface (represented by the triangular mesh), to avoid collision with the cutter during complex surface machining, it is essential to segment it into several patches. We transform the surface segmentation problem into a mathematical problem based on energy minimization according to several fabrication constraints, and solved by establishing a weighted graph and searching the minimum cut. Our algorithm has simple structure and is easy to implement. Moreover, the algorithm guarantees correctness and completeness in theory, that is, we prove that the weight of the minimum cut is equivalent to the minimum value of the energy function. Experimental results are provided to illustrate and clarify our method.

AMS subject classifications: 68U05, 68U07, 05C90

Key words: Surface segmentation, triangular mesh model, minimum cut, subtractive fabrication.

1 Introduction

High-end machining technology with high efficiency and high surface quality occupies a growing proportion in the complex workpiece manufacturing. A whole frame of subtractive fabrication includes computer aided design (CAD), computer aided manufacturing (CAM), computer numerical control (CNC) process. Nowadays with the development of industry, the practical CNC machining requires CAM having higher precision and higher efficiency. The tool path planning plays an essential part in CAM since the cutter must follow the path of the planned cutter location (CL) points in CNC machining.

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But in general, to machine a free-form 3D object in subtractive fabrication, it is impossible to finish the machining with only one single mounting posture of the object. So we first need to segment the object's surface into several patches so that each of which can be fully machined under one setup before the tool path planning. At present, the main processing methods of end milling includes 3-axis machining, 3+2-axis machining and 5-axis machining. We choose 3-axis/3+2-axis machining due to several factors. The 3-axis/3+2-axis machine has simple structure, which is easy to be operated. Compared to 5-axis machining, 3+2 machining can ensures similar accuracy but is more accessible and easier to work with. We use a ball-end cutter to machine the surface since the ball-end cutter is popularly used in industry.

With the rapid development of CAD/CAM technology in recent years, triangular mesh models have been widely used in the fields of discrete geometry modeling, CNC machining programming, graphics, and image. Because of the simplicity and maturity of the triangular mesh models, they have become the popular form in free-form surface representation in computer graphics and CNC machining. For a surface in the form of boundary representation such as Brep or a surface given discretely by cloud points, one can always generate a triangular mesh representation. Therefore, it is certainly meaningful to study the surface segmentation on the triangular mesh model. In this paper, we mainly focus on the semi-finishing milling, which is an essential process before the finishing machining. Usually, the semi-finishing surface to be machined is the envelope surface of the finishing process, but has fewer geometric features. So the relatively coarse meshes are our main process object.

There are different ways to deal with the surface segmentation in recent years. Luo *et al.* [10] took full account of the 3D printer's build volume constraint, and their segmented surfaces could be perfectly spliced together and leave no gaps. McCrae *et al.* [12] and Hildebrand *et al.* [8] segmented the shape into orthogonal plane patches that similar to the input model. Vanek *et al.* [18] divided the model into small pieces to save production time, and each piece strictly met the production volume constraint. Stava *et al.* [17] guaranteed the structural stability of prefabricated objects by using local deformation that satisfies the error constraint. Wang *et al.* [19] presented a method to segment a given model into several parts with graph cut such that their normals can be aligned perpendicularly to the printing direction. However, these methods mainly consider the problem in additive fabrication but not the constraints for subtractive fabrication. Mahdavi-Amiri *et al.* [11] introduced a novel carvable volume decomposition for efficient 3-axis CNC machining of 3D freeform objects, but their work is for rough machining. And Alderighi *et al.* [1] proposed a new method to decompose a 3D model into a set of double height fields that can be singularly fabricated with two-piece rigid casting. Unfortunately, their method does not apply to subtractive fabrication. Shen *et al.* [16] developed a new surface segmentation algorithm that can effectively preserve the sharp features using minimum spanning tree. But they did not consider the fabrication constraints.

We here give four suitable constraints to meet CNC subtractive fabrication including fabrication direction constraint, accessibility, patch number constraint and boundary con-