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Entropic Multi-Relaxation-Time Lattice Boltzmann Model for Large Density Ratio Two-Phase Flows

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Abstract. We propose a multiple relaxation time entropic realization of a recent two-phase flow lattice Boltzmann model [S.A. Hosseini, B. Dorschner, and I. V. Karlin, Journal of Fluid Mechanics 953 (2022)]. While the original model with a single relaxation time allows us to reach large density ratios, it is limited in terms of stability with respect to non-dimensional viscosity and velocity. Here we show that the entropic multiple relaxation time model extends the stability limits of the model significantly, which allows us to reach larger Reynolds numbers for a given grid resolution. The thermodynamic properties of the solver, using the Peng–Robinson equation of state, are studied first using simple configurations. Co-existence densities and temperature scaling of both the interface thickness and the surface tension are shown to agree well with theory. The model is then used to simulate the impact of a drop onto a thin liquid film with density and viscosity ratios matching those of water and air both in two and three dimensions. The results are in very good agreement with theoretically predicted scaling laws and experimental data.

AMS subject classifications: 7610, 76T10, 76D45

Key words: Lattice Boltzmann method, two-phase flows, entropic multiple relaxation time.

1 Introduction

Due to their presence in a wide range of applications, development of models for two-phase flows simulation holds an especially important place in any of the many numerical methods for computational fluid mechanics. The lattice Boltzmann method, developed in the early 90's is no exception to this general observation. Soon after the development of the first lattice Boltzmann models [28], extensions to two-phase flow physics were proposed [15, 35]. Over the past 30 years, a variety of formulations for two-phase

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flows, from the more classical Allen–Cahn and Cahn–Hilliard based formulations to the very popular pseudo-potential model [35], have been proposed and widely used. While routinely applied to many different configurations and used to model different physical phenomena, most of these approaches have struggled with large density ratios and high Reynolds number simulations [8, 25]. In the class of pseudo-potential and free energy formulations [27,38], these limitations appear in the form of deviations of the coexistence liquid/vapor densities from their analytical counter-parts at lower temperatures and are broadly referred to as thermodynamic inconsistency issues [8].

In a recent work [18], we proposed a kinetic scheme and a lattice Boltzmann realization exhibiting both thermo- and hydrodynamic consistency even at extremely high density ratios. It was shown that the scheme is not only suitable to capturing thermodynamic properties of the liquid-vapor interface and is thermodynamically well-posed but also allows for simulation of dynamic configurations at very high density ratios. However, relying on the simplest collision operator, i.e. single relaxation time, the simulations were limited in terms of the minimum non-dimensional viscosities and the maximum Courant–Friedrichs–Lewy (CFL) numbers. As a remedy, more advanced collision operators such as entropic [2, 3, 29], multiple relaxation time [10] and regularized [24] have been proposed. The multiple relaxation time collision operator has grown into the most widely used approach, for both single and two-phase flows. While effectively allowing for extended stability domains [13, 17, 23, 37, 42], it nevertheless lacks closures for the individual relaxation rates of the higher order moments. The entropic multiple relaxation model provides a physically motivated closure for the free parameter and in doing so allows for extended stability domains without tunable parameters [20]. A realization for two-phase flows based on a free-energy formulation was devised in [5].

Here, we propose a multiple relaxation time entropic realization of our previously proposed model [18] and thus increase the attainable Reynolds numbers. After a brief introduction of the model, it is first validated on simple configurations which probe thermodynamic properties such as coexistence densities at different temperatures and surface tension. Subsequently, simulations of drop impact on a liquid film are carried out first in two and then in three dimensions, at a density ratio of 10^3. The results show that the entropic multiple relaxation model provides a simple and effective means to overcome the stringent stability limits of the single relaxation time model.

2 Model description

The two-phase fluid is modeled using the continuum kinetic framework detailed in [18] and represented as,

$$\partial_t f + \boldsymbol{v}\cdot\boldsymbol{\nabla} f = -\frac{1}{\tau}\left(f - f^{\mathrm{eq}}\right) - \frac{1}{\rho}\frac{\partial f^{\mathrm{eq}}}{\partial \boldsymbol{u}}\cdot\left[\boldsymbol{\nabla}\left(P-P_0\right) - \kappa\rho\boldsymbol{\nabla}\boldsymbol{\nabla}^2\rho\right], \tag{2.1}$$

where f is the one-particle distribution function, $\boldsymbol{v}$ the particle velocity, κ the capillary coefficient in the second-gradient fluid model, ρ and $\boldsymbol{u}$ are the fluid density and velocity,