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A Characteristic Boundary Condition for Multispeed Lattice Boltzmann Methods

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Abstract. We present the development of a non-reflecting boundary condition, based on the Local One-Dimensional Inviscid (LODI) approach, for Lattice Boltzmann Models working with multi-speed stencils.
We test and evaluate the LODI implementation with numerical benchmarks, showing significant accuracy gains with respect to the results produced by a simple zero-gradient condition. We also implement a simplified approach, which allows handling the unknown distribution functions spanning several layers of nodes in a unified way, still preserving a comparable level of accuracy with respect to the standard formulation.

AMS subject classifications: 76M28, 76P05, 76M20

Key words: Characteristic boundary condition, non-reflective boundary conditions, local one-dimensional inviscid boundary conditions, lattice Boltzmann method, multispeed high order models.

1 Introduction

The Lattice Boltzmann Method (LBM) has emerged in the past decades as a computationally efficient fluid dynamic solver [1]. Originally developed for the simulation of isothermal weakly compressible flows, over the years several works have described possible approaches for extending the applicability of the method [2]. A possible pathway is offered by models based on higher order Gauss-Hermite quadrature [3–5], which in turn require the adoption of a discretization of the velocity space by means of multi-speed stencils. This approach has been successfully employed in the definition of LBM models for the study of, among others, compressible flows [3,6–8], rarefied gas flows [9–11] flows in curved space [12], semi-classic fluids [13] and relativistic flows [14].

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However, the presence of multiple speed levels introduces complications in the definition of accurate Boundary Conditions (BC). There is not much literature available in relationship to the development of boundary conditions for multi-speed LBM, with few examples of implementation of Dirichlet BC [15,16], diffusive BC [17], and the so called Tamm-Moth-Smith BC, specially devised for handling shock waves [7].

In this work, we take into consideration artificial BC, which are commonly employed for restricting a large, or even unbounded, physical domain to a (smaller) feasible computational domain. Ideally, the application of this class of BC should be such not to introduce spurious artifacts in the bulk dynamics. In particular, the BC should not cause reflections of pressure waves.

This type of setup is commonly modeled by employing a class of BC going under the name of Non-Reflecting BC (NRBC). A few examples are given by i) the perfectly matched layer technique [18], where a damping layer is attached to the computational domain, ii) the discrete artificial boundary condition [19], where the information entering the computational domain is approximated using another LBM simulation and iii) characteristic boundary conditions, where wave amplitude variations are manipulated (e.g. [20]). In this work we focus on this latter approach.

Based on the work by Hedstrom [21], Thompson [22] established characteristic boundary conditions for nonlinear hyperbolic systems such as the Euler equations. The general idea is to decompose information at the boundary into characteristic waves. The variation of the outgoing wave amplitude can be then computed from the adjacent fluid nodes, whereas incoming waves need to be specified using (application dependent) external information. This approach has been applied to non-hyperbolic Navier-Stokes equations by Poinsot and Lele [23]. In their work, the wave amplitude variations are approximated using the one-dimensional Euler equations. Dropping the transversal and viscous terms, this is referred to as a Local one-dimensional inviscid (LODI) problem.

The LODI procedure has been applied to single-speed LBM [20]. Thereby, macroscopic values obtained from the LODI problem are used to specify a Dirichlet condition in LBM simulations.

In this work, we extend the approach described in [20] to multi-speed models. We restrict our analysis to iso-thermal weakly compressible flows, in what can be seen as a first step towards the definition of a characteristic BC for high-order LBM capable of handling more complex flows.

This article is structured as follows: in Section 2 we provide a brief description of the multi-speed LBM used in this work. In Section 3, we give the mathematical formulation of the LODI framework, and introduce the extension to the multi-speed setting. We numerically evaluate the accuracy of this boundary condition for two benchmark cases, reporting the results in Section 4. Concluding remarks and future directions are summarized in Section 5.