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An Adaptive Non-Intrusive Multi-Fidelity Reduced Basis Method for Parameterized Partial Differential Equations

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Abstract. An adaptive non-intrusive multi-fidelity reduced basis method for parameterized partial differential equations is developed. Based on snapshots with different fidelity, the method reduces the number of high-fidelity snapshots in the regression model training and improves the accuracy of reduced-order model. One can employ the reduced-order model built on the low-fidelity data to adaptively identify the important parameter values for the high-fidelity evaluations under a given tolerance. The multi-fidelity reduced basis is constructed based on the high-fidelity snapshot matrix and the singular value decomposition of the low-fidelity snapshot matrix. Coefficients of such multi-fidelity reduced basis are determined by projecting low-fidelity snapshots on the low-fidelity reduced basis and using the Gaussian process regression. The projection method is more accurate than the regression method, but it requires low-fidelity snapshots. The regression method trains the Gaussian process regression only once but with slightly lower accuracy. Numerical tests show that the proposed multi-fidelity method can improve the accuracy and efficiency of reduced-order models.

AMS subject classifications: 65M10, 78A48

Key words: Multi-fidelity method, non-intrusive, reduced-order model, Gaussian process regression, adaptive sampling.

1. Introduction

Mathematical models described by parameterized partial differential equations (PDEs) attract substantial attention as substitutes for physical experiments in many scientific and engineering applications. Meanwhile, there are applications — e.g. uncertainty quantification and optimization design, which involve numerous evaluations of PDEs with different parameter values. However, the direct solution of large-scale systems is challenging work because of high computational costs and storage limitations. Therefore, constructing

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reduced-order models (ROMs) which provide an accurate and efficient approximation of the corresponding full-order models has become an important task — cf. [20, 30, 38, 39].

Here we focus on a very popular method in reduced-order modeling — viz. the reduced basis method (RBM) [4, 6, 9, 11, 45, 50]. Generally, RBM can be split into offline and online stages. In the offline stage, reduced basis functions are extracted from given high-fidelity snapshots by various reduction tools such as the proper orthogonal decomposition (POD) [3, 28]. In the online stage, the reduced order solution for a new parameter value can be restored by a linear combination of reduced bases. Therefore, an ideal selection of the reduced basis must satisfy the condition that the space expanded by the reduced basis is an accurate approximation to the solution space. Once a reduced basis is constructed, a classical intrusive approach — e.g. the Galerkin projection method, is often employed to determine the reduced coefficients [10, 22, 25, 41]. However, the intrusive methods are equation-dependent and the use of source codes is usually prohibited. Besides, the ROM approximation of nonlinear problems is not always stable [27]. Therefore, non-intrusive methods have been developed in different research areas [2, 13, 17, 18, 21, 35, 47].

The non-intrusive methods are equation-free data-driven approaches such that the governing equations are treated as black boxes. In these methods, the reduced coefficients are computed by interpolation or regression methods. In particular, Gaussian process regression (GPR) [15, 17, 18, 47] and artificial neural networks [13, 21] are ones of the most used regression methods, which perform well in reduced-order modeling. In these works, the mapping from the parameter space to the reduced coefficients is built by using high-fidelity data alone. However, in order to obtain acceptable results such an approach requires a large amount of high-fidelity data and requires heavy computational time. To mitigate this issue, many multi-fidelity models have been developed [23, 24, 29, 31–33, 37, 40, 49, 51, 53]. They are based on the fact that low-fidelity models are less accurate, but contain large-scale structures of the system. Therefore, one can reduce the requirement of high-fidelity data by using multi-fidelity data in reduced-order modeling. Thus, Kast *et al.* [23] proposed a multi-fidelity ROM based on a multi-fidelity GPR [5] with inputs from different levels of accuracy. The low-fidelity data are assimilated via an interpolation approach inspired by bi-fidelity reconstruction [33], which is a linear combination of high-fidelity snapshots. Lu and Zhu [29] presented a bi-fidelity data-assisted neural network in reduced-order modeling. The method generated the high-fidelity reduced basis using POD and learnt the high-fidelity reduced coefficients using a shallow multi-layer perception by incorporating the features extracted from the low-fidelity data as the input features. A multi-fidelity approach applied to physics-informed neural networks was proposed in [37].

We note that the finding of appropriate parameters is the key step in the construction of multi-fidelity ROMs since it can enhance the accuracy and efficiency. However, the traditional sampling methods of reduced-order modeling require a large number of sample points, especially in the case of high-dimensional parameter spaces. Therefore, it is important to establish a sampling method that can identify the important parameter values. The greedy sampling methods were introduced in [8, 14, 16, 19, 44, 44], where important parameter values have been sequentially selected from the parameters enabling the largest error of the ROM. Xiao *et al.* [46] used the sparse grid to generate important samples.