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Modeling Thermal Regulation in Thin Vascular Systems: A Mathematical Analysis

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Abstract. Mimicking vascular systems in living beings, designers have realized microvascular composites to achieve thermal regulation and other functionalities, such as electromagnetic modulation, sensing, and healing. Such material systems avail circulating fluids through embedded vasculatures to accomplish the mentioned functionalities that benefit various aerospace, military, and civilian applications. Although heat transfer is a mature field, control of thermal characteristics in synthetic microvascular systems via circulating fluids is new, and a theoretical underpinning is lacking. What will benefit designers are predictive mathematical models and an in-depth qualitative understanding of vascular-based active cooling/heating. So, the central focus of this paper is to address the remarked knowledge gap. *First*, we present a reduced-order model with broad applicability, allowing the inlet temperature to differ from the ambient temperature. *Second*, we apply mathematical analysis tools to this reduced-order model and reveal many heat transfer properties of fluid-sequestered vascular systems. We derive point-wise properties (minimum, maximum, and comparison principles) and global properties (e.g., bounds on performance metrics such as the mean surface temperature and thermal efficiency). These newfound results deepen our understanding of active cooling/heating and propel the perfecting of thermal regulation systems.

AMS subject classifications: 35B50, 35B51, 35Q79

Key words: Thermal regulation, vascular systems, reduced-order modeling, maximum and comparison principles, mathematical analysis, efficiency.

Principal notation

Symbol	Quantity
	Geometry-related quantities
$\mathcal{B}$	three-dimensional body
Ω	domain: mid-surface of the body

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$\partial\Omega$	boundary of the domain
Γ^{ϑ}	part of the boundary with prescribed temperature
Γ^{q}	part of the boundary with prescribed heat flux
Σ	curve denoting the vasculature
d	thickness of the body
$\widehat{\mathbf{n}}(\mathbf{x})$	unit outward normal vector at $\mathbf{x}$ on the boundary
s	arc-length along Σ measured from the inlet
$\widehat{\mathbf{t}}(\mathbf{x})$	unit tangential vector at $\mathbf{x}$ along the vasculature (Σ)
$\mathbf{x}$	a spatial point
	Solution fields
$\vartheta(\mathbf{x})$	temperature (scalar) field
$\mathbf{q}(\mathbf{x})$	heat flux (vector) field
	Prescribed quantities
ϑ_{amb}	ambient temperature
$\vartheta_{\mathrm{inlet}}$	temperature at the inlet
$\vartheta_{\mathrm{p}}(\mathbf{x})$	prescribed temperature on the boundary
$f(\mathbf{x})$	applied heat flux
f_0	constant applied heat flux
$\dot{m}$	mass flow rate in the vasculature
$q_{\mathrm{p}}(\mathbf{x})$	prescribed heat flux on the boundary
	Material and surface properties
ε	emissivity
c_f	heat capacity of the fluid flowing within the vasculature
h_T	convective heat transfer coefficient
$\mathbf{K}(\mathbf{x})$	thermal conductivity tensor field
	Other symbols
$[\![\cdot]\!]$	jump operator across the vasculature Σ
ϵ	symbol introduced to define a limiting process
η^{e}	efficiency
ϑ_{HSS}	hot steady-state temperature
$\vartheta_{\mathrm{mean}}$	mean temperature
σ	Stefan-Boltzmann constant $\approx 5.67\times10^{-8}$ [W/m^2/K^4]
χ	heat capacity rate of the fluid, $\chi=\dot{m}c_f$
$\mathrm{div}[\cdot]$	spatial divergence operator
$\mathrm{grad}[\cdot]$	spatial gradient operator
	Abbreviations
HSS	hot steady-state
PDE	partial differential equation