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A Five-Point TENO Scheme with Adaptive Dissipation Based on a New Scale Sensor

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Abstract. In this paper, a new five-point targeted essentially non-oscillatory (TENO) scheme with adaptive dissipation is proposed. With the standard TENO weighting strategy, the cut-off parameter C_T determines the nonlinear numerical dissipation of the resultant TENO scheme. Moreover, according to the dissipation-adaptive TENO5-A scheme, the choice of the cut-off parameter C_T highly depends on the effective scale sensor. However, the scale sensor in TENO5-A can only roughly detect the discontinuity locations instead of evaluating the local flow wavenumber as desired. In this work, a new five-point scale sensor, which can estimate the local flow wavenumber accurately, is proposed to further improve the performance of TENO5-A. In combination with a hyperbolic tangent function, the new scale sensor is deployed to the TENO5-A framework for adapting the cut-off parameter C_T, i.e., the local nonlinear dissipation, according to the local flow wavenumber. Overall, sufficient numerical dissipation is generated to capture discontinuities, whereas a minimum amount of dissipation is delivered for better resolving the smooth flows. A set of benchmark cases is simulated to demonstrate the performance of the new TENO5-A scheme.

AMS subject classifications: 35L65, 65M06, 76J20, 76N15, 76F05

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1 Introduction

For hyperbolic conservation laws, one of the most difficult issues is the development of high-order numerical schemes with the capability of capturing discontinuities sharply and preserving the high-order accuracy in smooth regions. The essentially non-oscillatory (ENO) [1] scheme has attracted lots of attention since it was proposed. Among a set of candidate fluxes, the ENO scheme selects the smoothest flux. Unlike ENO, the weighted ENO (WENO) scheme proposed by Liu et al. [2] uses a nonlinear convex combination of all candidate fluxes, including the non-smooth fluxes. This weighting strategy ensures the high-order accuracy in the smooth regions and the ENO property near discontinuities. After that, Jiang and Shu [3] propose the WENO5-JS scheme by introducing a new smoothness indicator and a novel finite-difference framework. However, further investigation demonstrates that the accuracy order of the WENO5-JS scheme degenerates near the critical points. To remedy this drawback, the WENO5-M [4] scheme remaps the weights calculated by WENO5-JS, and the WENO5-Z [5] scheme introduces a new weighting strategy by employing the global smoothness indicator. In addition, WENO5-JS generally produces excessive numerical dissipation, which may smear the small-scale structures in the flow field. The WENO-Z+ [6] scheme enhances the contribution of the less smooth candidate stencil flux to reduce the numerical dissipation. Recently, Sun et al. [7, 8] devise a method to optimize a class of finite difference schemes with the Minimized Dispersion and Controllable Dissipation (MDCD) properties by two independent parameters. More recently, Sun et al. [9] and Li et al. [10] present a finite difference scheme with minimum dispersion and adaptive dissipation (MDAD) properties by establishing a correlation between the local wavenumber and numerical dissipation. Different from altering the coefficients of the background linear schemes, the fourth- and fifth-order weighted compact nonlinear schemes (WCNS) [11] are developed by employing the compact schemes as the background schemes. The Hermite WENO (HWENO) [12] schemes are proposed based on the Hermite polynomials. Benefiting from the compactness of the reconstruction in these schemes, the three-point reconstruction can generate a fifth-order accuracy scheme. Furthermore, Cai et al. [13] apply the positivity-preserving techniques in the finite volume HWENO schemes for enhancing numerical stability. However, for the HWENO schemes, both the function values and the first-order derivatives need to be evolved in time and utilized in the reconstruction, which is nontrivial in terms of practical implementations. After that, Li et al. [14] introduce the multi-resolution HWENO schemes that only reconstruct the function values and obtain the first-order derivatives by the high-order linear polynomials. Overall, the main drawback of the compact schemes is that a global tridiagonal matrix needs to be solved at each time step, rendering them less efficient. Other variants include the central WENO (CWENO) schemes [15–20], the WENO-AO [21] and WENO-ZQ [22] schemes, the WENO scheme with automatic dissipation adjustment [23], and etc.

Different from the WENO weighting strategy, Fu et al. [24–28] propose a family of