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A Sample-Wise Data Driven Control Solver for the Stochastic Optimal Control Problem with Unknown Model Parameters

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Abstract. In this work, an efficient sample-wise data driven control solver will be developed to solve the stochastic optimal control problem with unknown model parameters. A direct filter method will be applied as an online parameter estimation method that dynamically estimates the target model parameters upon receiving the data, and a sample-wise optimal control solver will be provided to efficiently search for the optimal control. Then, an effective overarching algorithm will be introduced to combine the parameter estimator and the optimal control solver. Numerical experiments will be carried out to demonstrate the effectiveness and the efficiency of the sample-wise data driven control method.

AMS subject classifications: 93E11, 60G35, 65K10

Key words: Stochastic optimal control, parameter estimation, optimal filter, backward stochastic differential equations, stochastic gradient descent.

1 Introduction

In this paper, we develop an efficient online method for solving the data driven stochastic optimal control problem with unknown model parameters. In the classic stochastic optimal control problem, we want to find an optimal *"control process"* that controls a stochastic dynamical system (called the *"state process"*) to meet certain optimality conditions – such like minimizing a cost function. In many practical applications (e.g., smart grids [28, 33], nano-phase materials [14, 22, 31], and mathematical biology [8]), the controlled state model often contains unknown parameters. Therefore, we need to estimate

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the parameters first and then use the estimated parameters to design the optimal control. When parameter estimation and stochastic optimal control are combined together, the classic optimal control problem becomes the *"data driven optimal control"*, which is an important research topic that requires uncertainty quantification. The concept of "data driven" in our control problem refers to an extra parameter estimation procedure [25], and the estimated parameters will be the "driver" that guides the design of the optimal control. Extensive studies have been carried out to solve data driven optimal control problems with time-invariant linear controlled systems [1, 37, 43]. In this work, we will focus on data driven methods for more general *nonlinear* control problems.

As a key research topic in mathematical modeling, parameter estimation has been extensively studied. The efforts in solving the parameter estimation problem aim to collect observational data about model states and use the data information to estimate parameters. There are two major approaches to estimate unknown parameters: the optimization based point estimation and the Bayesian estimation [20, 30, 32, 42], and both approaches can be implemented in either the offline manner or the online manner. For the offline parameter estimation, we use all the observational data that we have ever collected to estimate the unknown parameters. On the other hand, online parameter estimation methods dynamically estimate and update the parameters based on the newly received data. Offline parameter estimation methods are well-studied, and they typically provide very accurate estimation results. However, the offline parameter estimation also has several disadvantages. First of all, when observations form a massive data set, large-scale data storage and analytics are needed, which often makes the offline parameter estimation prohibitive in practice. Especially, when a set of new observational data are received, offline methods need to go through all the historic data again to incorporate the incoming data, which can be very expensive. Secondly, although we often assume that the unknown parameters are static, it is also possible that the true values of the parameters change unexpectedly. For example, when estimating parameters that determine physical models in scientific experiments, the true values of the parameters may change unexpectedly due to sudden environmental changes or some unaware difference in the experimental set-up [14, 22]. In this case, offline parameter estimation methods are not suitable since the full data set may be generated by different true parameters.

The most widely used online parameter estimation method is the augmented filter [16, 24, 26]. The main idea of the augmented filter is to construct an augmented state-parameter process and let state estimation guide parameter estimation under the optimal filtering framework, which is a mathematical model that provides the best estimate for the state of a hidden stochastic dynamical system based on observational data [19, 29]. The drawback of the augmented filter is that the indirect parameter estimation strategy may not always be efficient enough in terms of usage of data. In fact, most effort in the optimal filtering procedure in the augmented filter would contribute to state estimation since the observations only describe the state, and the data only give direct evidence on the accuracy of state estimation.

In this work, we introduce a novel *direct filter* method to estimate the unknown param-