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An Iterative Thresholding Method for the Minimum Compliance Problem

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Abstract. In this paper, we propose a simple energy decaying iterative thresholding algorithm to solve the two-phase minimum compliance problem. The material domain is implicitly represented by its characteristic function, and the problem is formulated into a minimization problem by the principle of minimum complementary energy. We prove that the energy is decreasing in each iteration. Two effective continuation schemes are proposed to avoid trapping into the local minimum. Numerical results on 2D isotropic linear material demonstrate the effectiveness of the proposed methods.

AMS subject classifications: 65K10, 74B05, 74P15, 35K08

Key words: Minimum compliance, convolution, thresholding, linear elasticity.

1 Introduction

Topology optimization as a design tool has attracted increasing attention recently due to the improvement of computing power and manufacturing technology. The last decade has seen a great amount of work in this important area of structural optimization [4, 18, 28, 42]. This has mainly been spurred by the success of the additive manufacturing (such as 3D printing) for generating optimal topologies of structural elements. The technology is now well established, and designs obtained with topology optimization methods are

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in production on a daily basis. The aerospace industry and the automotive industry, for example, apply sizing and shape optimization to the design of structures and mechanical elements. Shape optimization is also used in the design of electrochemical, acoustic, and electromagnetic devices [5, 12, 24]. The optimization of the geometry and topology of structural layouts has great impact on the performance of structures.

There are numerous techniques to perform a topology optimization [27,29] since the original work by Bendsøe and Kikuchi [7] on the homogenization approach. The most popular method in the field is the Solid Isotropic Material with Penalization (SIMP) technique which is based on determining the optimum structure within the predefined domain by allowing the material density to take values between 0 and 1 and interpolating the stiffness with power functions [6]. An alternative scheme is the Rational Approximation of Material Properties(RAMP) [30] which uses a family of rational functions to interpolate the stiffness. SIMP and RAMP are said to have similar performance [26], though SIMP appeared earlier and has since gained popularity due to its simplicity and efficiency. Other methods include level set method approach [2, 37, 38], phase field method [9, 32, 41], bubble method [14, 15], evolutionary structural optimization (ESO) approaches [39] and several others [1,7]. For a comparative review, see [29].

Topology optimization finds boundaries among different phases while threshold dynamics is an efficient method for solving constraint optimization problems involving domains and interfaces. First developed by Merriman, Bence and Osher (MBO) [23], the threshold dynamics method represents the interface implicitly by characteristic functions and simulates curvature motion of the interface via alternating two simple steps: diffusion and thresholding. In [16], it was pointed out that each iteration corresponds to minimizing a linear relaxation of a concave objective functional on a convex set. Its simplicity and unconditional stability have attracted applications in grain growth simulation of polycrystalline materials [13], image segmentation [17, 21, 34, 35], optimal partitioning [22,31,33], wetting dynamics [36,40], and topology optimization for fluids [10,19].

In [10], an efficient iterative method for topology optimization for fluids modeled with the Stokes equation was proposed based on the threshold dynamics method. The problem can be solved with an iterative scheme in which the Stokes equation is approximated by a Brinkman equation and the characteristic function describing the fluid-solid regions is updated according to simple convolution followed by a thresholding step. It was shown that the iterative algorithm has the total energy decaying property. The proposed algorithm is simple and easy to implement, and the numerical results show that the algorithm is very robust and insensitive to the initial guess and the parameters in the model.

Topology optimization of continuum structures aims to optimize the distribution of material within a defined domain by fulfilling given constraints and minimizing a predefined cost function. One of the most important and simplest problems is the minimum compliance problem with a material volume constraint. In this paper, we develop a threshold dynamics method for the minimum compliance problem. In the method, the minimum compliance problem is formulated in terms of stresses based on the princi-