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Influence of the Radial Inertia Effect on the Propagation Law of Stress Waves in Thin-Walled Tubes

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Abstract. This investigation focused on the influence of the radial inertia effect on the propagation behavior of stress waves in thin-walled tubes subjected to combined longitudinal and torsional impact loads. Generalized characteristics theory was used to analyze the main features of the characteristic wave speeds and simple wave solutions in thin-walled tubes. The incremental elastic-plastic constitutive relations described by the rate-independent plasticity were adopted, and the finite difference method was used to investigate the evolution and propagation behaviors of combined elastic-plastic stress waves in thin-walled tubes when the radial inertial effect was considered. The numerical results were compared with those obtained when the radial inertia effect was not considered. The results showed that the speed of the coupled stress wave increased when the radial inertia effect was considered. The hardening modulus of the material in the plastic stage had a greater impact on the coupled slow waves than on the coupled fast waves.

AMS subject classifications: 74J30

Key words: Radial inertia effect, thin-walled tube, stress waves, finite difference method, characteristic theory.

1 Introduction

When a dynamic load is applied on thin-walled tubes, stress waves propagate through the tubes, and the tubes subsequently deform. The dynamic behaviors of thin-walled tubes are complicated, because the stresses and strains in the tubes depend on the properties of the material, the tube shapes and dimensions, the duration of the load, and other factors. As a typical engineering structure, thin-walled tubes have low weights and costs and can be easily processed. They are widely used in marine engineering, transportation,

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aerospace, defense engineering, and other fields. Therefore, it is of great significance to understand the propagation behaviors of stress waves in thin-walled tubes subjected to impact loads.

The problem of combined elastic-plastic stress wave propagation in thin-walled tubes was first investigated by Clifton [1]. He predicted an initial decrease in the shear strain in a thin-walled tube statically pre-torsioned to the plastic range when a longitudinal impact was applied on it. Soon after, Lipkin and Clifton [2, 3] conducted experiments in which annealed aluminum tubes were subjected to static plastic torques followed by longitudinal compressive impacts. The experimental results qualitatively verified the theoretical predictions. Goel and Malvern [4] studied the same problem for a combined kinematic and isotropic hardening material. They drew the conclusion that the hardening characteristics could make a significant difference in the results. Ting [5, 6] obtained solutions for all possible combinations of discontinuous loadings at $t=0$ and for all possible variations of the loads thereafter. Wu and Lin [7] applied the theory of plasticity without a yield surface and obtained a simple wave solution of a thin-walled tube subjected to combined step loading. The numerical examples of their theoretical analysis were in good agreement with the experiments conducted by Lipkin and Clifton [2, 3]. Based on self-consistent slip models for the independent, isotropic, latent hardening of face-centered cubic metals, solutions were compared with experimental results reported previously for the case of aluminum tubes subjected to static torques followed by an axial impact [8]. The model predictions showed better agreement with experiments than predictions from theories based on plasticity models. In addition, studies based on plastic flow theory investigated the propagation behavior of stress waves in a thin-walled tubes made of elastic-plastic-viscoplastic materials [9], phase transformation materials [10–12], and functionally graded materials [13], and some unusual phenomena were observed.

The work mentioned above was based on the assumption that the mean radius of the tube was much larger than the tube wall thickness, ignoring the influence of the radial inertial effect. It is reasonable to expect that under rapid dynamic loading, the radial inertia effect would play a significant role in the propagation of stress waves along a thin-walled tube. Mayes and Eisenberg [14, 15] investigated the influence of the radial inertia effect on the propagation behavior of stress waves in thin-walled tubes. They found by numerical calculations that the propagation speed and amplitude of the combined stress waves changed when the radial inertial effect was considered. Li et al. [16] used the incremental elastic-plastic constitutive relations and numerical methods to study the evolution and propagation behaviors of combined stress waves in a thin-walled tube considering the radial inertia effect. The numerical results showed that the radial inertia of the tube had a greater effect on the propagation of combined stress waves. The above research found that the radial inertia effect had an influence on the propagation behaviors of the stress wave in the thin-walled tube, but the mechanism behind this influence needs to be clarified through further analysis.

The main purpose of this study was to further investigate how the radial inertia effect influences the propagation and evolution of combined stress waves in thin-walled tubes