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On G-quotient Mappings and Networks Defined by G-convergence

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Abstract. Let G_1,G_2 be methods on topological spaces X and Y respectively, $f:X\to Y$ be a mapping, $\mathscr{P}$ be a cover of X. f is said to be a (G_1,G_2)-quotient mapping provided $f^{-1}(U)$ is G_1-open in X, then U is G_2-open in Y. $\mathscr{P}$ is called a G-cs'-network of X if whenever $\boldsymbol{x}=\{x_n\}_{n\in\mathbb{N}}\in c_G(X)$ and $G(\boldsymbol{x})=x\in U$ with U open in X, then there exists some $n_0\in\mathbb{N}$ such that $\{x,x_{n_0}\}\subset P\subset U$ for some $P\in\mathscr{P}$. $\mathscr{P}$ is called a G-kernel cover of X if $\{(U)_G:U\in\mathscr{P}\}$ is a cover of X. In this paper, we introduce the concepts of (G_1,G_2)-quotient mappings, G-cs'-networks and G-kernel covers of X, and study some characterizations of (G_1,G_2)-quotient mappings, G-cs'-networks, and G-kernel covers of X. In particular, we obtain that if G is a subsequential method and X is a G-Fréchet space with a point-countable G-cs'-network, then X is a meta-Lindelöf space.

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Key words: G-methods, G-convergence, G-quotient mappings, G-cs'-networks, G-kernel covers, G-Fréchet spaces.

1 Introduction

Convergence of sequences in a topological space is a basic and important concept in mathematics. In addition to the usual convergence of sequences, statistical convergence, ideal convergence and even the general G-convergence have attracted extensive attention [1–3]. Based on several kinds of convergence properties in real analysis, Connor and Grosse-Erdmann [1] introduced G-methods defined on a linear subspace of the vector space of all real sequences, G-convergence on real spaces and G-continuity for real functions, studied the relationship among G-continuous functions, linear functions and continuous functions, established the dichotomy theorem of G-continuity and extended several known results in the literature. Çakallı [4–6] extended the concepts to topological

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groups satisfying the first axiom of countability, and defined G-sequential compactness and G-sequential connectedness. At the same time, he also discussed G-sequential continuity by means of G-sequential closures and G-sequentially closed sets.

As we know, mappings and networks are important concepts in investigating topological spaces. Continuous mappings, quotient mappings , pseudo-open mappings, *cs*-networks, *sn*-networks, *k*-networks and so on are the most important tools for studying convergence, sequential spaces, Fréchet-Urysohn spaces and generalized metric spaces [7]. For this reason, this paper draws into (G_1,G_2)-quotient mappings, (G_1,G_2)-covering mappings, G-cs'-networks and G-kernel covers, and discusses some basic properties of them.

Recently, Mucuk and Şahan [8] have introduced the notions of G-sequentially open sets and G-sequential neighborhoods of first-countable topological groups and investigated G-sequential continuity in topological groups. Liu [9, 10] gave some properties of G-neighborhoods, G-continuity at a point, G-derived sets and G-boundaries of a set. Mucuk and Çakallı [11] have introduced the G-Connectedness in topological groups. Wu and Lin [12] have introduced the properties of G-connectedness and G-topological groups. Ping and Liu [13] have introduced the properties of G-connectedness in generalized topology spaces.

Let X be a set, $s(X)$ denote the set of all X-valued sequences, i.e., $x\in s(X)$ if and only if $\boldsymbol{x}=\{x_n\}_{n\in\mathbb{N}}$ is a sequence with each $x_n\in X$. If $f:X\to Y$ is a mapping, then $f(\boldsymbol{x})=\{f(x_n)\}_{n\in\mathbb{N}}$ for each $\boldsymbol{x}=\{x_n\}_{n\in\mathbb{N}}\in s(X)$. If X is a topological space, the set of all X-valued convergent sequences is denoted by $c(X)$, and we put $\lim\boldsymbol{x}=\lim_{n\to\infty}x_n$ for any $\boldsymbol{x}\in c(X)$. $\langle x_n\rangle$ denotes the subset $\{x_n:n\in\mathbb{N}\}$ of X. In this paper, all topological spaces are assumed to satisfy the T_2 separation property and all mappings are surjection. The readers may refer to [7, 14] for notation and terminology not explicitly given here.

2 Preliminaries

Definition 2.1. *Let X be a set.* (1) *A method on X is a function $G:c_G(X)\to X$ defined on a subset $c_G(X)$ of $s(X)$. A sequence $\mathbf{x}=\{x_n\}_{n\in\mathbb{N}}$ in X is said to be G-convergent to $l\in X$ if $\mathbf{x}\in c_G(X)$ and $G(\mathbf{x})=l$.*

(2) *Let X be a topological space.*

(2.1) *A method $G:c_G(X)\to X$ is called regular if $c(X)\subset c_G(X)$ and $G(\mathbf{x})=\lim\mathbf{x}$ for each $\mathbf{x}\in c(X)$.*

(2.2) *A method $G:c_G(X)\to X$ is called subsequential if, whenever $\mathbf{x}\in c_G(X)$ is G-convergent to $l\in X$, then there exists a subsequence $\mathbf{x}'\in c(X)$ of $\mathbf{x}$ with $\lim\mathbf{x}'=l$.*

(2.3) *A method $G:c_G(X)\to X$ is called point if, whenever the constant sequence $\mathbf{x}=\{x,x,x,\cdots\}\in c_G(X)$ with $G(\mathbf{x})=x$ for each $x\in X$.*

By Definition 2.1, we see that the definitions of G-methods and G-convergence do not involve a topology of a set X. Obviously, statistical convergence [2] method in topological spaces is a regular method; admissible ideal convergence [3] method in topological