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An Alternating Direction Method of Multipliers for Inverse Lithography Problem

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Abstract. We propose an alternating direction method of multipliers (ADMM) to solve an optimization problem stemming from inverse lithography. The objective functional of the optimization problem includes three terms: the misfit between the imaging on wafer and the target pattern, the penalty term which ensures the mask is binary and the total variation regularization term. By variable splitting, we introduce an augmented Lagrangian for the original objective functional. In the framework of ADMM method, the optimization problem is divided into several subproblems. Each of the subproblems can be solved efficiently. We give the convergence analysis of the proposed method. Specially, instead of solving the subproblem concerning sigmoid, we solve directly the threshold truncation imaging function which can be solved analytically. We also provide many numerical examples to illustrate the effectiveness of the method.

AMS subject classifications: 78A46, 78M50

Key words: Inverse lithography techniques, ADMM framework, total variation regularization.

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1. Introduction

Optical lithography system plays a critical role in semiconductor industry. The optical lithography system can print mask patterns onto the wafer through an optical system. As shrinkage of integrated circuit device size and subject to the resolution limit of the optical system, the diffraction of mask makes the patterns on wafer distorted very much. To remedy that, many resolution enhancement techniques are proposed and used extensively in optical lithography. Optical proximity correction (OPC) is the most popular resolution enhancement method. With optical proximity correction, people adjust the mask layout such that the output pattern approximates the desired one. OPC can be divided into two categories: rule based OPC and model based OPC. Rule

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based method modifies the mask layout and compensates for the warping in local features empirically, which is easy to implement. Model based method relies on the physical and mathematical theories of optical imaging. With model based OPC, the mask pattern is represented by a pixel-wise function or a level-set function, then the OPC process is modeled as an inverse problem. Usually, the inverse problem is formulated as a non-convex optimization problem about the mask pattern. So the model based OPC is usually called inverse lithography technology (ILT) [26]. The objective functional of this inverse problem is the misfit between the image on wafer and the target pattern. The image on wafer is modeled by threshold and truncation of aerial image projected by the optical system and the threshold is used to mimic the exposure process of photoresist on wafer. By solving the inverse problem, ILT can adjust the shape of mask such that the mask gets close to the optimal one. It is known that the aerial image of mask is approximated by Abbe formula which can be regarded as convolution [14]. Besides, the threshold function is discontinuous, then the objective functional of inverse lithography is non-smooth and non-convex, and traditional optimization algorithms are difficult to achieve good results.

Since the importance of the inverse lithography problem, tremendous efforts have been devoted to solve the problem in the past decades. Based on the pixel-wise mask, the inverse lithography problem was stated as nonlinear, constrained minimization problems, and linear, quadratic, and nonlinear formulations of the objective function were considered in [15]. The steepest descent methods were proposed in [20, 27]. A conjugate gradient method was suggested to accelerate the computation in [20]. And there are many other works related to the gradient-based method such as [3, 4, 18, 29, 31]. The level set method is another effective way to solve inverse lithography problem. With this method, the target pattern, mask pattern, and wafer pattern are represented by level set functions, then the inverse lithography problem is regarded as a shape and topology optimization problem [32, 33]. There are many other miscellaneous methods in inverse lithography, such as the genetic algorithm [11], the route of particle swarm optimizer combined with the adaptive nonlinear control strategy [35], deep convolution neural network methods [40, 41]. A fuller review of inverse lithography techniques could be found in [25].

It is known that the inverse lithography problems are ill-posed. Regularization method is an efficient way to remedy the ill-posedness [10]. The regularization framework was proposed to control the tone and complexity of the synthesized masks [30], L^1-regularization, wavelet and total-variation (TV) regularizations are discussed in [18]. Sparsity and low-rank regularization terms are used to constrain the solution space and reduce the mask complexity. The split Bregman algorithm is used to solve the inverse optimization problem with sparsity (L^1 norm) and low rank (nuclear norm) regularizations [21]. Specially, the TV regularization is much attractive due to its good edge-preserving property [34]. An objective functional that consists of several total-variation regularization terms is introduced in [9], where the total variations about mask and image on wafer are both considered. But the TV regularization is not differentiable and it is difficult to implement. Some methods are proposed to deal with the