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An Efficient Nonlinear Multigrid Solver for the Simulation of Rarefied Gas Cavity Flow

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Abstract. We study efficient simulation of steady state for multi-dimensional rarefied gas flow, which is modeled by the Boltzmann equation with BGK-type collision term. A nonlinear multigrid solver is proposed to resolve the efficiency issue by the following approaches. The unified framework of numerical regularized moment method is first adopted to derive the high-quality discretization of the underlying problem. A fast sweeping iteration is introduced to solve the derived discrete problem more efficiently than the usual time-integration scheme on a single level grid. Taking it as the smoother, the nonlinear multigrid solver is then established to significantly improve the convergence rate. The OpenMP-based parallelization is applied in the implementation to further accelerate the computation. Numerical experiments for two lid-driven cavity flows and a bottom-heated cavity flow are carried out to investigate the performance of the resulting nonlinear multigrid solver. All results show the wonderful efficiency and robustness of the solver for both first- and second-order spatial discretization.

AMS subject classifications: 76P05, 65N55

Key words: Boltzmann equation, moment method, multigrid, rarefied gas flow, steady state.

1 Introduction

Rarefied gas dynamics is a classical discipline of fluid mechanics, which studies gas flows in the situation when the molecular mean free path, compared with the characteristic length of problems, can not be negligible. Over the past several decades, it has attracted

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more and more attention in the high-tech fields including astronautics and micro-electromechanical systems, due to the rapid development of aerospace and semiconductor industries, etc. To describe the rarefied gas flows correctly, the Boltzmann equation, rather than the traditional continuum models such as Euler equations and Navier-Stokes equations, should be taken into account. Clearly, numerical methods are necessary to solve the Boltzmann equation. Because of the intrinsic high dimensional nature, it encounters a big challenge to develop an accurate and efficient solver for the Boltzmann equation, especially for a great of important purposes [30,42] when the steady state of problems has to be under investigation. Many instructive algorithms have been developed under the framework of discrete velocity method and Fourier spectral method, see e.g. [33, 37, 40], to resolve the efficiency issue.

This work is concerned with efficient simulation of steady state for rarefied gas flows modeled by the Boltzmann equation with simplified relaxation models, e.g., the Bhatnagar-Gross-Krook (BGK) model [1], in the framework of numerical regularized moment method originated in [9] and then developed in [6, 10–12]. As one of the most powerful methods, the moment method derives a system of continuum equations, which can be viewed not only as an extended hydrodynamic model but also a high-order velocity discretization of the Boltzmann equation. The numerical regularized moment method enhances this system with a slight modification so that some desired properties, such as global hyperbolicity [5] and convergence [4, 14], could be well guaranteed for the revised moment system up to arbitrary order. Moreover, in the framework of numerical regularized moment method, the full discretization would be obtained formally by using the finite volume method for spatial discretization without explicitly writing out the moment system. Such a unified framework makes the practical application of high-order moment system much easier. Additionally, both first- and second-order spatial discretization are of interest in the present paper.

To efficiently solve the derived discrete steady-state problem, a fast sweeping iteration, based on the forward Euler scheme and the cell-by-cell Gauss-Seidel iteration with four alternating direction sweepings in two-dimensional (2D) case, is first proposed. Benefiting from the fact that each sweeping direction covers a family of characteristics of the problem so that the important characteristic property could be utilized, as mentioned in [29, 38], the present fast sweeping iteration would converge much faster, in comparison to the forward Euler scheme. Although each step of the former takes almost the same computational cost as four steps of the latter, it turns out that the fast sweeping iteration is usually more efficient than the forward Euler scheme. Indeed, it can also be observed that the fast sweeping iteration is more robust than the forward Euler scheme especially when the second-order spatial discretization is under consideration.

The multigrid method [3, 18, 34] is one of the most popular acceleration techniques for steady-state computation. It has been successfully applied to the gas flow studies even with high-order schemes, such as the p-multigrid compact gas-kinetic scheme (CGKS) [27], and the multigrid unified gas-kinetic scheme (UGKS) [39, 43]. In view of these, we also concentrate on the multigrid acceleration for our discrete problem. Actu-