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Analysis of Discontinuity Detectors and Hybrid WCNS Schemes Based on Waveform Recognition

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Abstract. In this paper, we present a hybrid form of weighted compact nonlinear scheme (WCNS) for hyperbolic conservation laws by applying linear and nonlinear methods for smooth and discontinuous zones individually. To fulfill this algorithm, it is inseparable from the recognition ability of the discontinuity detector adopted. In specific, a troubled-cell indicator is utilized to recognize unsmooth areas such as shock waves and contact discontinuities, while avoiding misjudgments of smooth structures. Some classical detectors are classified into three basic types: derivative combination, smoothness indicators and characteristic decomposition. Meanwhile, a new improved detector is proposed for comparison. Then they are analyzed through identifying a series of waveforms firstly. After that, hybrid schemes using such indicators, as well as different detection variables, are examined with Euler equations, so as to investigate their ability to distinguish practical discontinuities on various levels. Simulation results demonstrate that the proposed algorithm has similar performances to pure WCNS, while it generally saves 50 percent of CPU time for 1D cases and about 40 percent for 2D Euler equations. Current research is in the hope of providing some reference and establishing some standards for judging existing discontinuity detectors and developing novel ones.

AMS subject classifications: 65M06, 65M12, 35L65, 35L04
Key words: WCNS, waveform detection, troubled-cell indicator, shock-capturing, hybrid scheme.

1 Introduction

Supersonic compressible flow is described by Euler or Navier-Stokes equations. Through decades of development in computational fluid dynamics (CFD), methods for its numerical solution have been extensively studied by a good many scholars. When the flow

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gets disturbed, abundant structures will be generated correspondingly, including both smooth parts such as turbulent vortices in various scales, and discontinuities like shocks. Shock waves can lead to sudden changes for flow variables, consequently becoming a key point in the simulation. During the solution, it is necessary to not only maintain high fidelity for resolving small-scale details, but also apply sufficient numerical dissipation to suppress spurious oscillations. As a result, under above contradictory requirements, it has become a major challenge to construct high-order schemes with high resolution, so as to accurately simulate complex flow fields.

In order to meet these demands, WCNS effectively combines approaches for smooth and discontinuous zones. In specific, Lele [1] first proposed a compact scheme whose resolution is close to spectral method, but only fitting linear schemes. To solve this problem, Deng and Maekawa [2] put forward CNS (Compact Nonlinear Scheme). Then Deng and Zhang [3] applied nonlinear weighting technique and get WCNS, which shows nice shock-capturing property. After that, Deng et al. [4] pointed out that the resolution of WCNS is dominated by interpolation. Along with this thought, Zheng et al. [5] projected a self-adaptive algorithm namely WCNS-ε to further enhance fidelity. Zheng also indicated that large gradient zones generating from high-order critical points, would give rise to oscillations unless the mesh is fine enough, so they need to be dealt as discontinuities. What's more, WCNS shows vitality in many other aspects. Zhu et al. [6] constructed another type of hybrid WCNS for integrated solution on structured and unstructured mesh by combining CPR method. He et al. [7] extended time discretization of WCNS through adopting two-stage fourth-order approach.

On the other hand, WENO (Weighted Essentially Non-Oscillatory) approximation provides another approach. This shock-capturing scheme was proposed by Liu et al. [8], and was furtherly improved by Jiang and Shu [9]. By judging the level of smoothness in substencils, it can adaptively select nonlinear weights, so as to sharply capture shock waves. However, due to the use of nonlinear weighting technique, WENO scheme actually has difficulty in fully degenerating to its linear counterpart in the smooth area [10]. This gives rise to nonnegligible numerical dissipation and results in the smearing of small-scale structures.

There are commonly two kinds of methods to improve dispersion and dissipation properties for weighted nonlinear schemes: enhancing nonlinear mechanism, and constructing hybrid schemes. In this paper, the latter category is discussed. Due to its simplicity in operation, the application of hybrid scheme has been studied with increasingly more researchers. By dividing discontinuous zones from smooth regions, it can make better use of either shock-capturing schemes or low dissipation high-fidelity ones, thus possessing higher efficiency compared to a single algorithm with full characteristic decomposition [11]. However, before dealing with smooth areas and discontinuities separately, it is essential to be able to identify both of them correctly. Fortunately, discontinuity detectors are just effective means of such recognition. At present, there has developed so many types of troubled-cell indicators that the choice produces great impact on resolution and robustness of the scheme. Therefore, the comparison and selection of detectors