

An Iterative Thresholding Method for the Heat Transfer Problem

Luyu Cen¹ and Xiaoping Wang^{1,2,3,*}

¹ *Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China*

² *School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen, Guangdong 518172, China*

³ *Shenzhen International Center for Industrial and Applied Mathematics, Shenzhen Research Institute of Big Data, Shenzhen, Guangdong 518172, China*

Received 24 May 2023; Accepted (in revised version) 27 July 2023

Dedicated to the memory of Professor Zhongci Shi

Abstract. In this paper, we propose a simple energy decaying iterative thresholding algorithm to solve the heat transfer problem. The material domain is implicitly represented by its characteristic function, and the problem is formulated into a minimum-minimum problem. We prove that the energy is decreasing in each iteration. Numerical experiments for two types the heat transfer problems (volume to point and volume to sides) are performed to demonstrate the effectiveness of the proposed methods.

AMS subject classifications: 80M50, 74B05, 74P15

Key words: Optimization in heat transfer, convolution, thresholding.

1 Introduction

Topology optimization has been widely applied to the design of heat transfer systems to improve their performance while minimizing material usage and manufacturing

*Corresponding author.

Emails: lcen@connect.ust.hk (L. Cen), mawang@ust.hk (X. Wang)

costs [1, 7, 14, 19, 21]. Thermal management of electronic components involves controlling their temperature to ensure that they operate within safe limits and perform optimally. Electronic components generate heat during operation, and if the heat is not dissipated efficiently, it can lead to premature failure or reduced performance. Effective thermal management is critical for the performance and reliability of electronic components, particularly in high-power applications such as data centers [18], power electronics [15], and electric vehicles [17, 20].

Topology optimization methods can be used to design heat sinks and other cooling devices that are highly optimized for thermal management. These methods can help to improve the performance and efficiency of electronic components and other thermal management systems, while also reducing their size and weight. There are several different topology optimization methods that can be used for thermal management, including density-based methods which use a density function to represent the material distribution within the heat transfer devices and update the density function iteratively to optimize the material distribution for maximum heat transfer efficiency [2, 3, 8, 11, 24]; level set methods which use a level set function to represent the geometry of thermal conductive material [13, 25]. See [6, 10] for reviews.

Among the various methods used for topology optimization, the threshold dynamics method [9, 16] has recently gained attention due to its ability to handle complex and nonlinear problems with high computational efficiency. The threshold dynamics method is a mathematical framework for topology optimization based on iteratively updating a characteristic function which separates the design domain into two regions. The method has been successfully applied to various engineering problems, including image segmentation [22, 23], fluid channel design [5], flow network design [12], minimum compliance problem [4], by optimizing the characteristic function to achieve a desired performance objective.

In this paper, we present a novel application of the threshold dynamics method to topology optimization of heat transfer systems. We focus on the implementation of the method and its performance in optimizing the thermal performance of heat transfer systems. Specifically, we demonstrate the effectiveness of the method in improving the heat transfer rate and reducing thermal resistance by optimizing the topology of heat transfer components.

To achieve this, we start by introducing the conductive steady-state heat transfer problem defined by Bejan [2]. The problem represents an electrical device that is cooled down by a limited amount of high conductive material aiming at driving the produced heat to a heat sink, located at the boundary of the finite size volume. We then formulate the optimization problem in terms of characterization function of the domain and design a threshold dynamics method to solve the problem. We present the results of our numerical simulations, demonstrating the effectiveness of