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SEMI-IMPLICIT SPECTRAL DEFERRED CORRECTION METHODS BASED ON SECOND-ORDER TIME INTEGRATION SCHEMES FOR NONLINEAR PDES*

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Abstract

In [20], a semi-implicit spectral deferred correction (SDC) method was proposed, which is efficient for highly nonlinear partial differential equations (PDEs). The semi-implicit SDC method in [20] is based on first-order time integration methods, which are corrected iteratively, with the order of accuracy increased by one for each additional iteration. In this paper, we will develop a class of semi-implicit SDC methods, which are based on second-order time integration methods and the order of accuracy are increased by two for each additional iteration. For spatial discretization, we employ the local discontinuous Galerkin (LDG) method to arrive at fully-discrete schemes, which are high-order accurate in both space and time. Numerical experiments are presented to demonstrate the accuracy, efficiency and robustness of the proposed semi-implicit SDC methods for solving complex nonlinear PDEs.

Mathematics subject classification: 65M60, 35L75, 35G25.
Key words: Spectral deferred correction method, Nonlinear PDEs, Local discontinuous Galerkin method, Second-order scheme.

1. Introduction

Following the method-of-lines approach, the application of the local discontinuous Galerkin (LDG) method for spatial discretization of PDE will generate a system of ODEs. In some cases, the right hand side can be written as the sum of two terms, a stiff one (F_S) and a non-stiff one (F_N)

$$\begin{cases} u_t = F_S(t, u(t)) + F_N(t, u(t)), \quad t \in [0, T], \\ u(0) = u_0. \end{cases} \tag{1.1}$$

An efficient time discretization technique to solve the above ODEs is semi-implicit methods [10, 26], which treating the non-stiff terms explicitly and the stiff terms implicitly.

However, not all ODEs containing stiff and non-stiff components appear in partitioned form (1.1), and therefore the use of standard semi-implicit schemes is not straightforward. Boscarino *et al.* [2] developed a new semi-implicit Runge-Kutta method to solve a large class of PDEs

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and obtained high-order accuracy. However, the Runge-Kutta method has some limitations, for example, it is much more difficult to construct for higher order accuracy. Motivated by these, we developed a semi-implicit SDC method [20] to solve ODEs without easily separating of stiff and non-stiff components.

Dutt, Greengard and Rokhlin first developed a variation on the classical defect or deferred correction methods [1,21], called the SDC method [11]. It is based on first-order time integration methods, which are corrected iteratively, with the order of accuracy increased by one for each additional iteration. Then, a semi-implicit SDC method was introduced by Minion [29] to solve equations containing both stiff and non-stiff components. Recently, the semi-implicit SDC method was generalized for solving a series of nonlinear problems [14, 19, 28, 34, 35], which all have easily separating of stiff and non-stiff components. In the semi-implicit SDC scheme, one treats the stiff components implicitly and the non-stiff components explicitly. Various numerical simulations demonstrate that the semi-implicit SDC method is effective and robust.

The original SDC methods are based on first-order time integration methods, which are corrected iteratively, with the order of accuracy increased by one for each additional iteration. In [3,4,7–9], a variant of SDC, integral deferred correction (IDC), constructed using uniform nodes and high-order Runge-Kutta integrators in both the prediction and corrections was introduced. Using a Runge-Kutta method of order r in the correction results in r more degrees of accuracy with each successive correction. It was demonstrated that the IDC methods are more efficient than SDC methods based on first-order time integration methods. Motivated by the idea, we will develop a class of semi-implicit SDC methods, which are based on second-order time integration methods and the order of accuracy are increased by two for each additional iteration. In addition, the SDC methods are efficient for a large class of PDEs, including those without easily separating stiff and non-stiff components.

To use the SDC methods, the main difficulty is to construct an efficient basic second-order scheme that is unconditionally stable. One idea is based on the second-order Crank-Nicolson/Adams-Bashforth (CN/AB) method, the other one is based on the second-order invariant energy quadratization (IEQ) approach which will result in high-order linear schemes. Many application problems, such as the convection diffusion equation, the surface diffusion of graphs, the nonlinear Schrödinger equation and the gradient flow models, can be solved by using the proposed SDC scheme coupled with high-order LDG methods.

The rest of this paper is organized as follows. In Section 2, we develop two SDC schemes, which are based on the second-order CN/AB method, and the IEQ approach respectively. In Section 3, we present some applications of the proposed SDC methods, including the convection diffusion equation, the surface diffusion of graphs, the nonlinear Schrödinger equation, the Allen-Cahn equation, the Cahn-Hilliard equation and the Cahn-Hilliard phase field model of the binary fluid-surfactant system. Numerical examples are also given to validate the proposed SDC methods. Finally, we give the concluding remarks in Section 4. In Appendix A, we give the analysis of the accuracy for the SDC method. In Appendix B, we take the nonlinear Schrödinger equation as an example to illustrate the LDG method and to prove the energy stability in the fully-discrete level.

2. SDC Schemes Based on Second-Order Time Integration Methods

In this section, we will develop a class of SDC methods, which are based on second-order time integration methods and the order of accuracy are increased by two for each additional iteration.