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Uncertainty Comparison Between Value-at-Risk and Expected Shortfall

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Abstract. Value-at-Risk (VaR) and expected shortfall (ES) are two key risk measures in financial risk management. Comparing these two measures has been a hot debate, and most discussions focus on risk measure properties. This paper uses independent data and autoregressive models with normal or t-distribution to examine the effect of the heavy tail and dependence on comparing the nonparametric inference uncertainty of these two risk measures. Theoretical and numerical analyses suggest that VaR at 99% level is better than ES at 97.5% level for distributions with heavier tails.

AMS subject classifications: 62P05, 62E20

Key words: α-mixing, asymptotic variance, expected shortfall, Value-at-Risk.

1 Introduction

Using economically meaningful risk measures is vital in market regulation, portfolio management, and the banking and insurance industry. Two popular risk measures are Value-at-Risk (VaR) and expected shortfall (ES). The Value-at-Risk

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has been adopted for measuring market risk in trading portfolios since 1990. Because of its lack of subadditivity and insensitivity to extreme losses, Artzner *et al.* [3] advocate the coherent expected shortfall risk measure. In 2016, the minimum capital requirement for market risk in the recent revision by the Basel Committee on Banking Supervision (BCBS) had moved from Value-at-Risk at 99% level to expected shortfall at 97.5% level to capture more extreme risks (Danielsson and Zhou [9]). The reason to use different risk levels is that the difference between these two risk measures is tiny when the loss has the standard normal distribution.

Comparing these two risk measures has been hot and intensive in the literature. Emmer *et al.* [11] compare the pros and cons of Value-at-Risk and expected shortfall and argue that expected shortfall is better in practice, despite some shortcomings regarding its estimation backtesting. Embrechts *et al.* [10] discuss from risk aggregation and model uncertainty viewpoint and provide a broadly accessible critical assessment of the Value-at-Risk and expected shortfall debate triggered by Basel III. Because Cont *et al.* [8] argue that robustness is as vital as the coherence properties, Kou *et al.* [16] compare these two risk measures using robustness related to model misspecification and tiny changes in data. Krätschmer *et al.* [17] compare a list of risk measures, including Value-at-Risk and expected shortfall by the index of qualitative robustness. Gneiting [14] shows that ES is not elicitable, while Fissler and Ziegel [13] show that ES is jointly elicitable with VaR.

In this paper, we theoretically and empirically examine the effect of heavy tails and serial dependence on comparing the nonparametric inference efficiency of the Value-at-Risk at 99% level and the expected shortfall at 97.5% level. A related but different study is Barnard *et al.* [4], where they compare the nonparametric inference efficiency using independent observations with exponential power distributions. Our main conclusion is that using VaR at 99% level is better than ES at 97.5% level in terms of nonparametric inference efficiency when the underlying loss distribution has a heavier tail. This conflicts with the preference of using ES as it is argued that ES takes more extremes into account.

We organize the paper as follows. Section 2 presents our theoretical and numerical comparison results. Section 3 is a simulation study to confirm our findings in Section 2. Section 4 analyzes two insurance datasets. Section 5 concludes.

2 Theoretical and numerical comparisons

For a random variable X representing the loss of a financial institution or risk variable, the Value-at-Risk and expected shortfall at risk level $p \in (0,1)$ are de-