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Splitting Physics-Informed Neural Networks for Inferring the Dynamics of Integer- and Fractional-Order Neuron Models

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Abstract. We introduce a new approach for solving forward systems of differential equations using a combination of splitting methods and physics-informed neural networks (PINNs). The proposed method, splitting PINN, effectively addresses the challenge of applying PINNs to forward dynamical systems and demonstrates improved accuracy through its application to neuron models. Specifically, we apply operator splitting to decompose the original neuron model into sub-problems that are then solved using PINNs. Moreover, we develop an L^1 scheme for discretizing fractional derivatives in fractional neuron models, leading to improved accuracy and efficiency. The results of this study highlight the potential of splitting PINNs in solving both integer- and fractional-order neuron models, as well as other similar systems in computational science and engineering.

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1 Introduction

The human brain is a complex system that involves the interactions of billions of neurons. Mathematical models can be used to simulate the neuronal activity in the brain as a system of differential equations, allowing researchers to better understand how the brain works. Studies related to spiking neurons are performed numerically or biophysically. In numerical studies, the main goal is to solve neural equations and investigate how the dynamic behavior changes for different inputs. Biophysical approaches focus on interpreting the dynamic behavior of spiking neurons according to available experimental observations [2,61].

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Another interesting aspect of spiking neuron models is that they can be formulated as fractional-order equations, which take into account long-term memory. The order of the derivative in these equations can affect the neuron's response [57, 62, 66], making this an important area of research. Recent works in both integer- and fractional-order neuron models are discussed in Section 3.3.

In this work we introduce a new approach for solving neuron models that combines operator splitting methods with physics-informed neural networks (PINNs). Operator splitting methods have been successfully applied in various fields of physics and engineering [7,12,16,19,25,36,54,55], while PINNs provide a powerful tool for approximating the solution of differential equations. A general introduction to the splitting method can be found in [27, 46].

PINNs were first introduced by Raissi et al. [53]. In this method, the solution of a differential equation is approximated using a neural network, and the parameters of the network are determined by solving a minimization problem that includes residual functions at collocation points, as well as initial and boundary conditions.

PINNs have been applied successfully to a broad range of ordinary and partial differential equations, including fractional equations [51], integro-differential equations, stochastic partial differential equations [71], and inverse problems [47]. There have also been several extensions to the original PINN, such as Fractional PINN (FPINN) [51], physics-constrained neural networks (PCNN) [38, 72], variable hp-VPINN [32], conservative PINN (CPINN) [31], Bayesian PINN [69], parallel PINN [58], Self-Adaptive PINN [45], and Physics informed Adversarial training (PIAT) [56]. Some other recent works can be found in [6, 24, 42]. Innovations in activation functions, gradient optimization techniques, neural network structures, and loss function structures have driven recent advances in the field. Despite these advances, improvements are still possible, especially concerning unresolved theoretical and practical issues.

Our study makes two important contributions to the field of neural modeling. First, we propose a new method, called the splitting PINN, that employs the operator splitting technique to decompose the original spiking neuron model into sub-problems, which are then solved using PINNs. We demonstrate the effectiveness and accuracy of this method by applying it to integer- and fractional-order neuron models with oscillatory responses, for which vanilla PINN and FPINN formulations fail to predict the solutions. Second, we introduce a novel L^1-scheme for discretizing fractional derivatives in fractional neuron models, which leads to improved accuracy and efficiency in solving these complex models. Our results show that the combination of the splitting PINN method and the L^1-scheme accurately solves fractional neuron models and provides valuable insights into the underlying mechanisms of neural activity.

This paper is organized as follows: Section 2 provides an overview of the proposed method for a given system of differential equations. Section 3 introduces various neuron models and their properties, including the Leaky Integrate-and-Fire (LIF), Izhikevich, Hodgkin-Huxley (HH), and the fractional order Hodgkin-Huxley (FO-HH) models. The efficiency and accuracy of splitting PINNs and FPINNs are demonstrated in Section 4